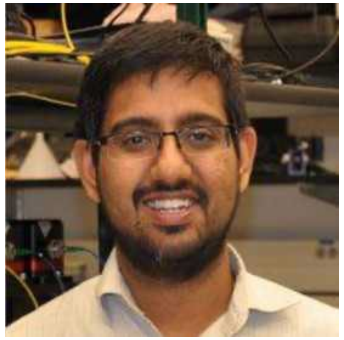
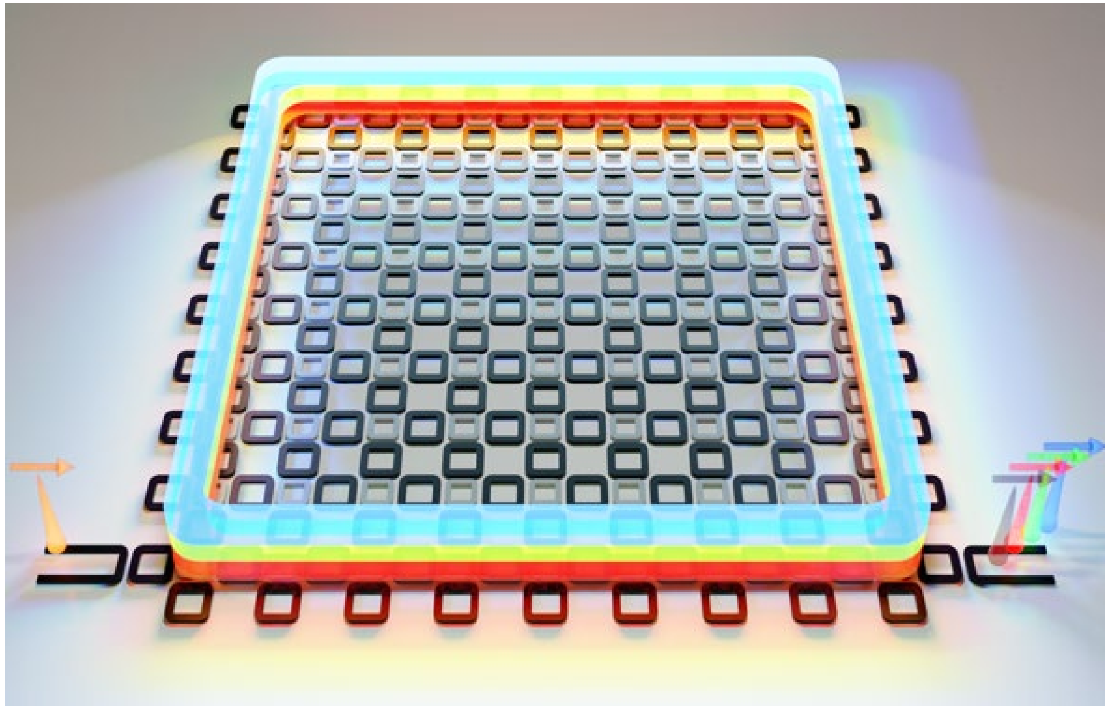
Integrated nonlinear topological photonics

Mohammad Hafezi



Kartik Srinivasan



JOINT QUANTUM INSTITUTE

UNIVERSITY OF MARYLAND

Speaker: Lida Xu
Graduate student @ Hafezi group

PQE, Snowbird, Jan 2026

M.J. Mehrabad†, L Xu†, M. Hafezi* et al. Science (2025)
C. Flower†, M.J. Mehrabad†, L Xu†, M. Hafezi* et al. Science (2024)

Outline

- Concept and history
  - Topological photonics
  - Integrated optical frequency combs
- Integrated nonlinear topological photonics
  - **(Theory + Exp)** Topological Frequency Combs (TFC)
  - **(Exp)** Multi-timescale mode locking of TFC
  - **(Exp)** Nested phase matching for high yield nonlinear optics
  - **(Theory + Exp)** The dispersive tight-binding framework.
  - **(Theory)** Exotic nonlinear solutions with Quantum Metamorphosis

Topology is physical!
Can topology emerge with neutral particles?

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Robert B. Laughlin
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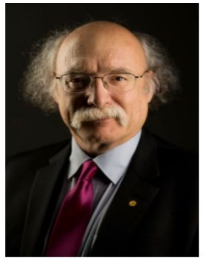

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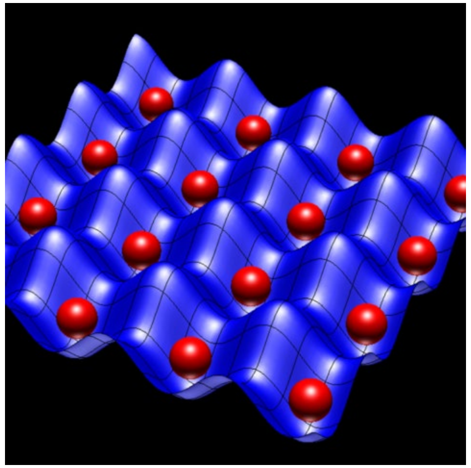
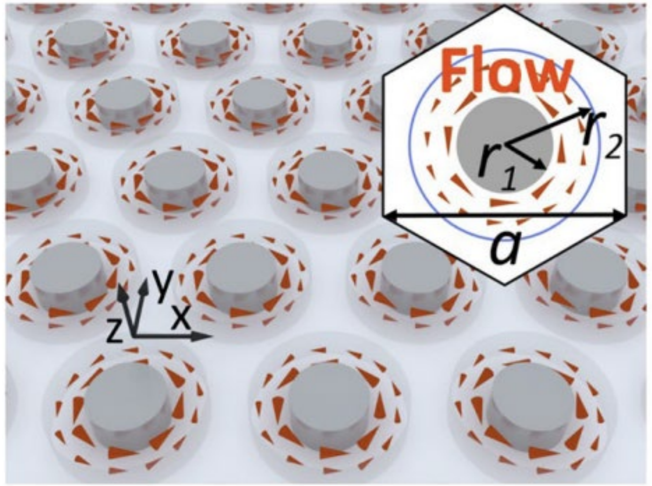
1985: Integer Quantum Hall effect
1998: Fractional Quantum Hall effect
2016: (Haldane) Anomalous Quantum Hall effect

Atoms



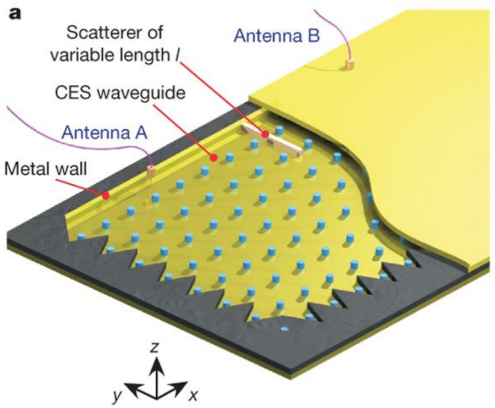
M Hafezi. PRA (2007)
(Image: NIST)

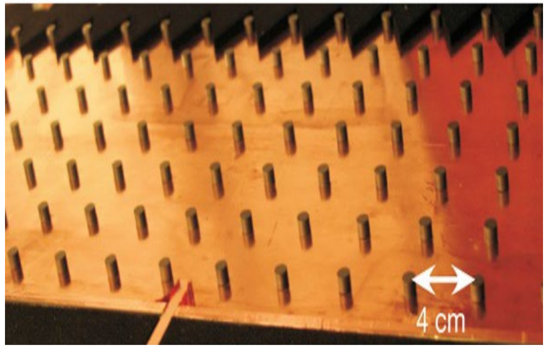
Phonons



ZJ Yang, BL Zhang et al. PRL (2008)

Microwave (wavelength 1 mm - 1 m)





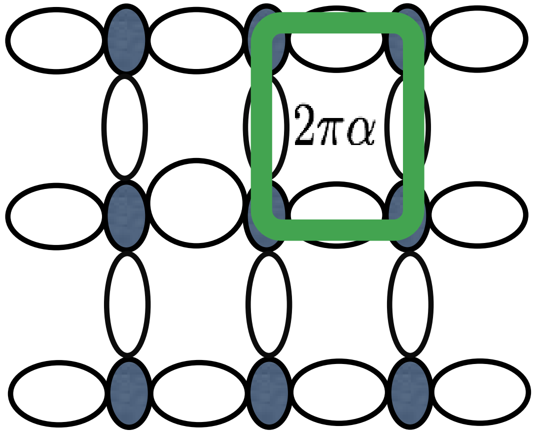
Haldane and Raghu. PRL (2008)
Wang Zheng, Marin Soljacic, et al. Nature (2009)

How about optical domain? (wavelength: 200 nm – 2500 nm)

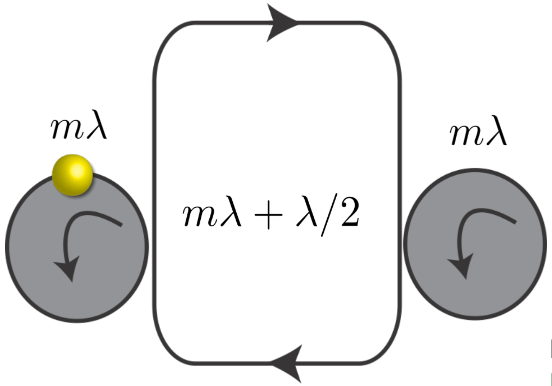
Photons don’t interact with magnetic fields!

Artificial Gauge Fields needed!

➢ Artificial Gauge Field in micro-resonators



Two resonator case:



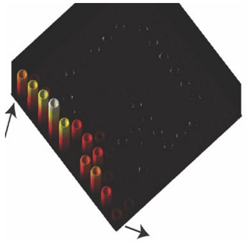
Integer Quantum Hall effect (IQH)

$$H_{IQH} = -J\sum_{x,y}(\hat{a}^{\dagger}{}_{x+1,y}\hat{a}_{x,y}\,e^{-iy\phi} + \hat{a}^{\dagger}{}_{x,y+1}\hat{a}_{x,y} + h.c.)$$

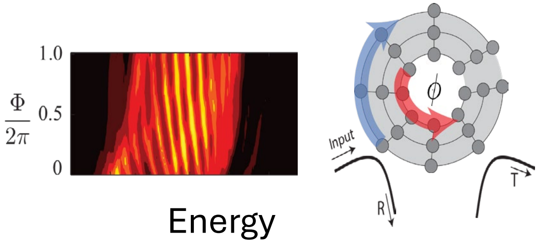
Anomalous Quantum Hall effect (AQH)

$$H_{AQHE} = -J(\sum_{\langle m,n\rangle} \hat{a}^{\dagger}{}_{m}\hat{a}_{n}\,e^{-iy\phi_{m,n}} + \sum_{\langle\langle m,n\rangle\rangle} \hat{a}^{\dagger}{}_{m}\hat{a}_{n} + h.c.)$$

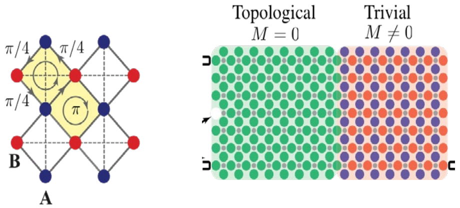
Imaging photonic topological edge states
Nature Photonics 7, 1001 (2013)



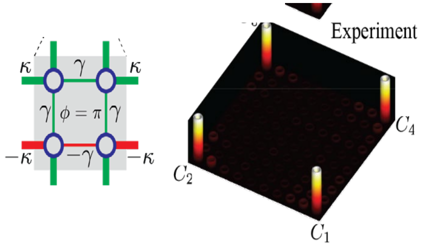
Demonstration of spectral flow/Chern number
PRL 112, 210405 (2014)
Nature Photonics 10, 180 (2016)



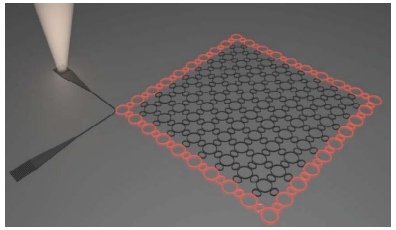
Photonic Quantum Anomalous Hall effect
PRL 123, 043201 (2019)



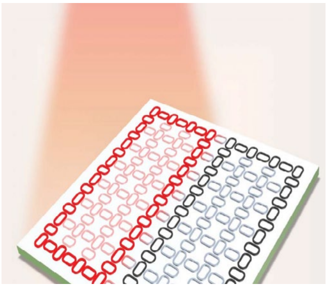
Photonic quadruple topological phases
Nature Photonics 13, 692 (2019)



Topological laser
Khajavikhan & Segev's group Science 359 (2018)



Reconfigurable topological states
Feng group (UPenn) Science 365, 1163 (2019)



For waveguides see Rechtsman et al. Nature (2013)

2005: optical frequency combs
How? Phase-stabilized, mode-locked femtosecond lasers.

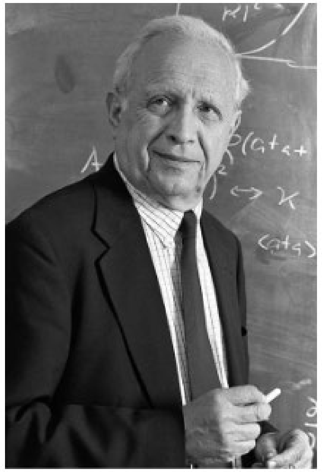
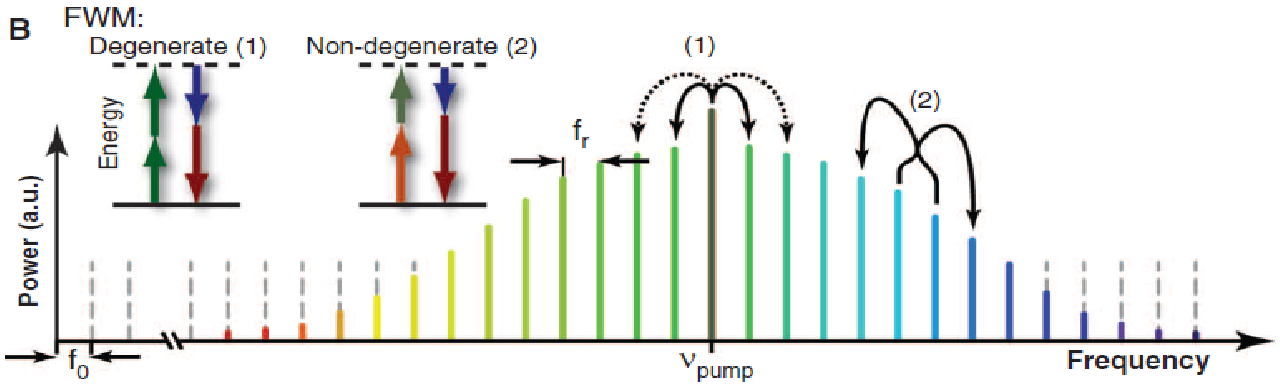


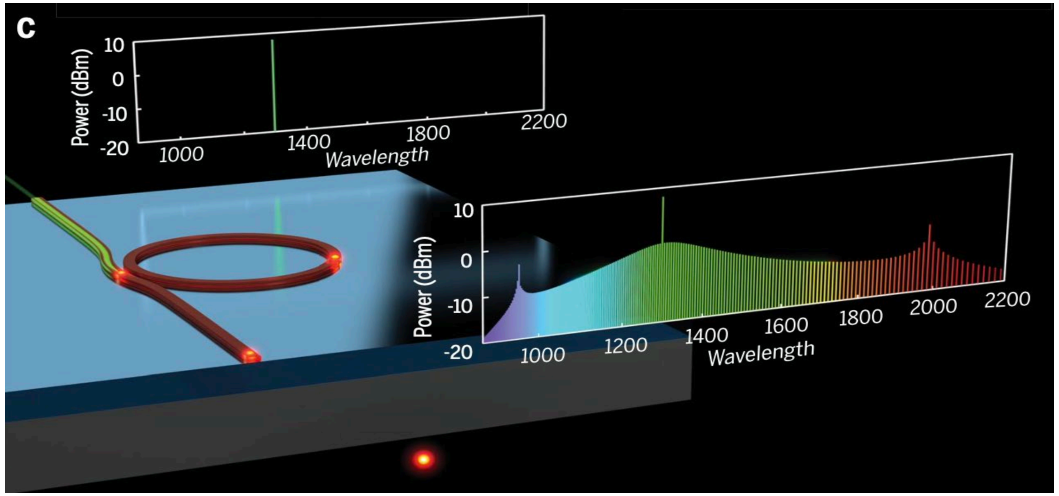
Photo: J.Reed
Roy J. Glauber
Prize share: 1/2

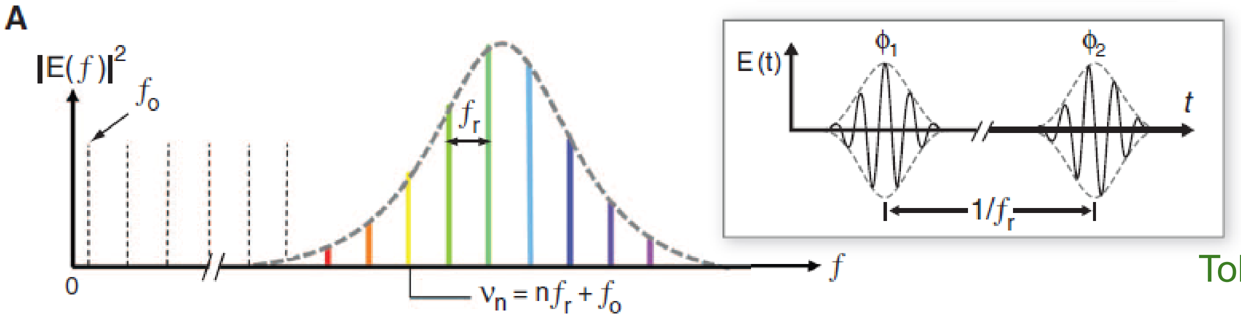
Photo: Sears.P.Studio
John L. Hall
Prize share: 1/4

Photo: F.M. Schmidt
Theodor W. Hänsch
Prize share: 1/4

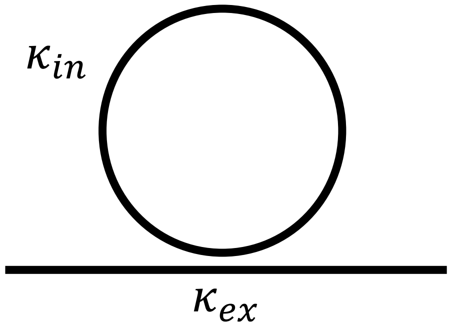
Can we miniaturize the frequency comb generator ?







Tobias J. Kippenberg, Ronald Holzwarth, and Scott A. Diddams. Science (2011)
Tobias J. Kippenberg, Ronald Holzwarth, and K.J.Vahala. Science (2016)
Tobias J. Kippenberg, Alexander L. Gaeta, Michal Lipson, Michael L. Gorodetsky. Science (2018)



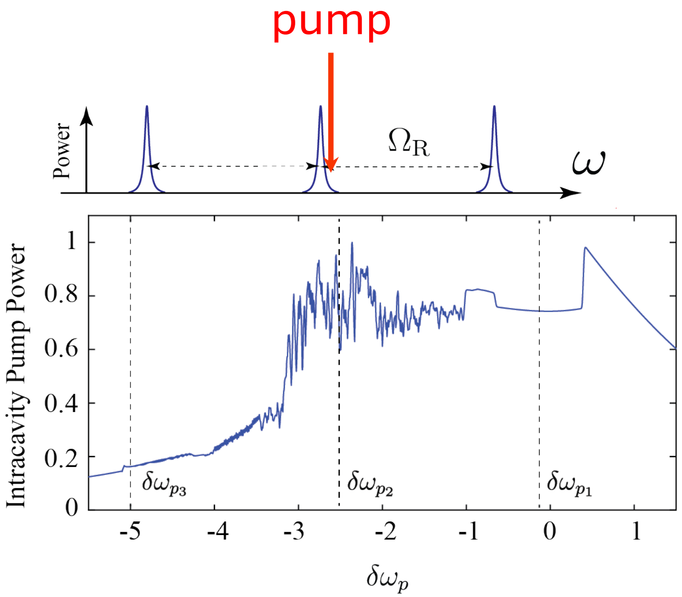
Theoretical description of microcombs: Lugiato–Lefever Equation (LLE)
A nonlinear Schrödinger equation (NLSE)

$$\mathbf{A(\theta, t)} = \sum_{\mathbf{l}} A_l(t) e^{\mathbf{i}\left(\omega_l - \omega_{l_0}\right)\mathbf{t} - \mathbf{i}(\mathbf{l} - l_0)\theta}$$

$$\frac{\partial}{\partial t} A_l = -(\kappa_{in} + \kappa_{ex}) A_l + \mathbf{i}\sigma A_l + \sqrt{2\kappa_{ex}}\mathbf{F} + \mathbf{i}g_0 |A_l|^2 A_l$$

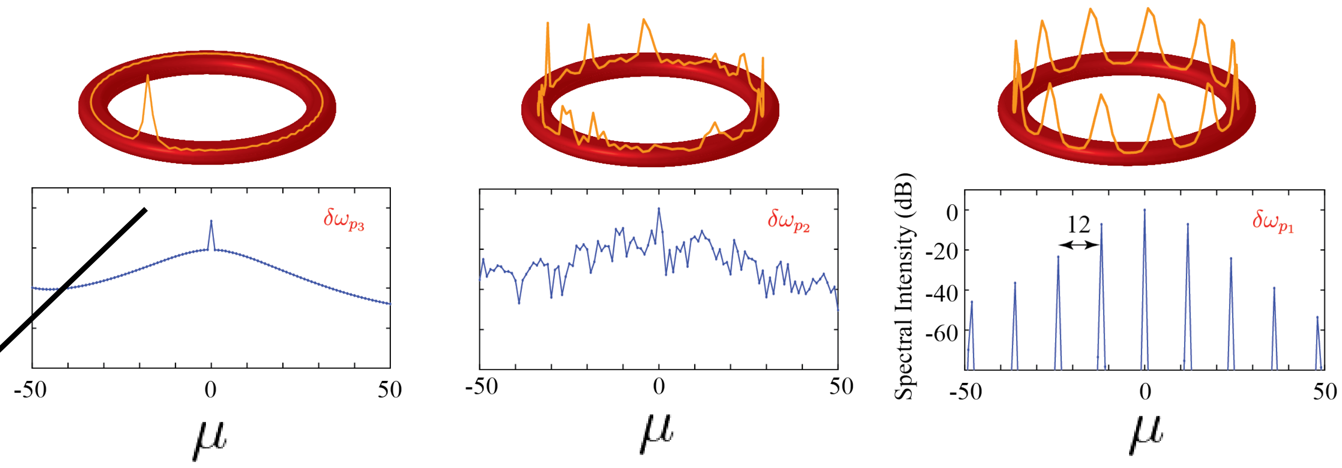
$\sigma = \omega_l - \omega_{l_0}$, **detuning**

Different nonlinear states arises from Kerr frequency combs, depending on different parameters and sweeping techniques.



power distribution

Comb envelope

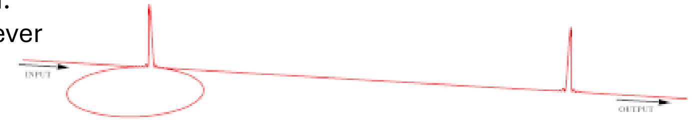


Soliton

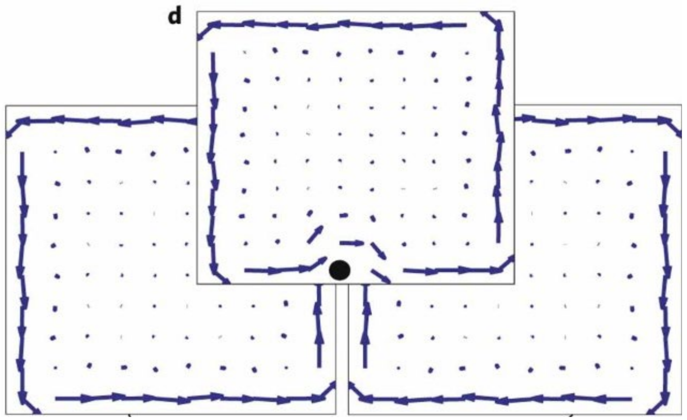
Chaotic

Turing Roll

Wikipedia: Lugiato–Lefever equation



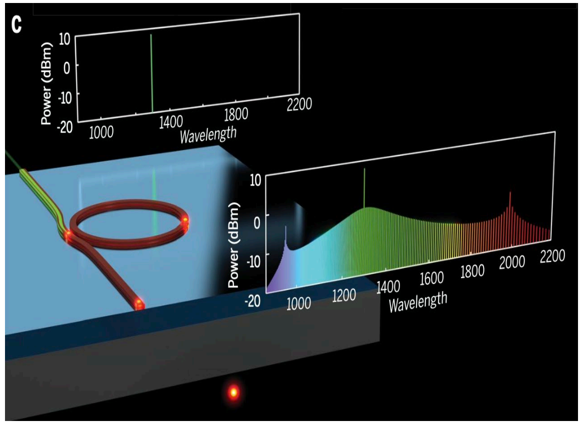
Yanne Chembo and Curtis Menyuk. PRA (2013)

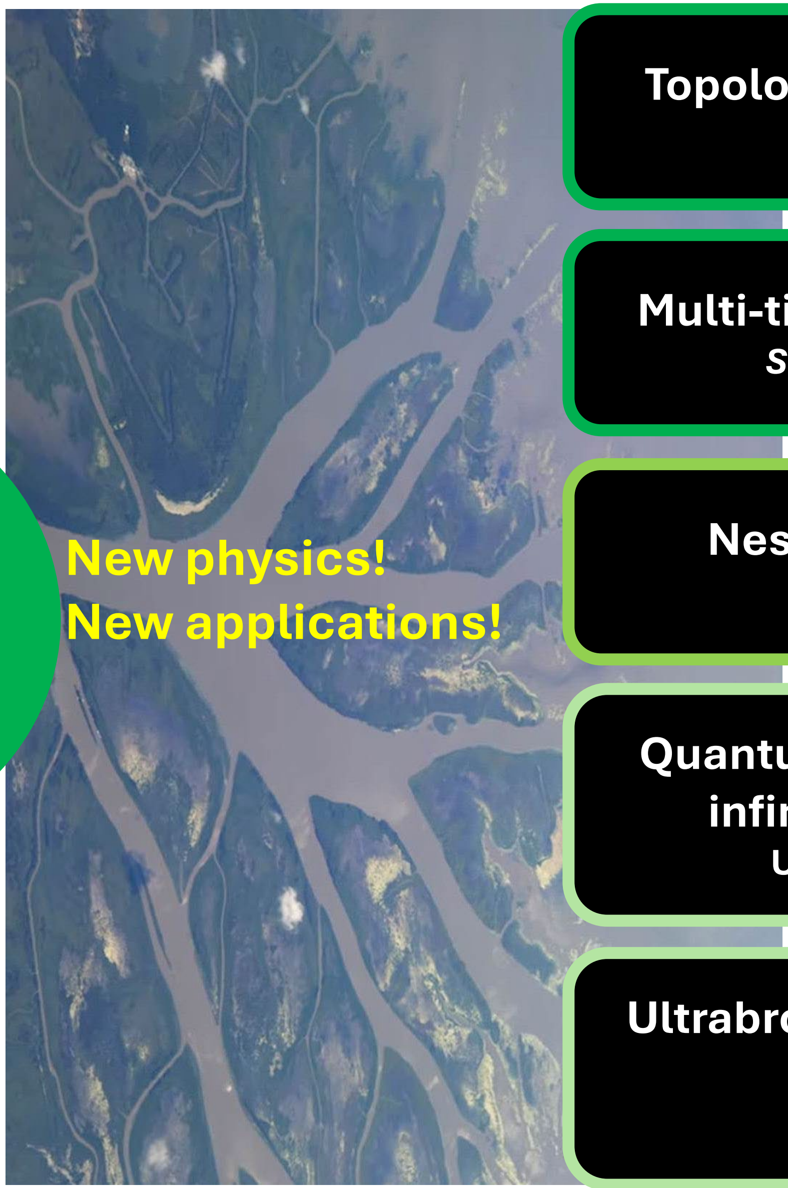


Topological photonics

Topological Nonlinear Photonics

Micro combs





New physics!
New applications!

Topological frequency combs
Science 2024

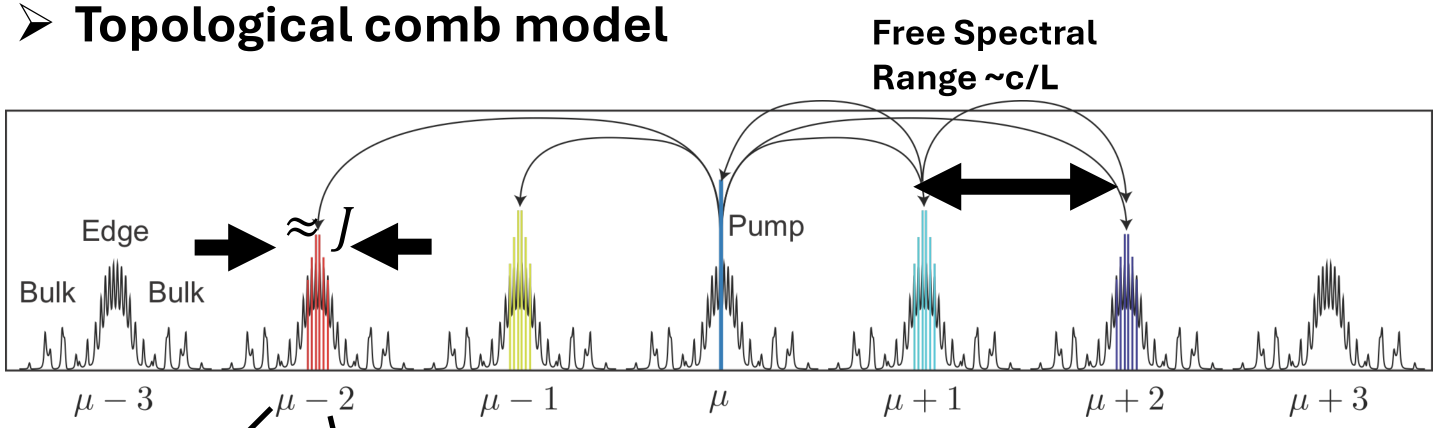
Multi-timescale mode-locking
Science Advances 2025

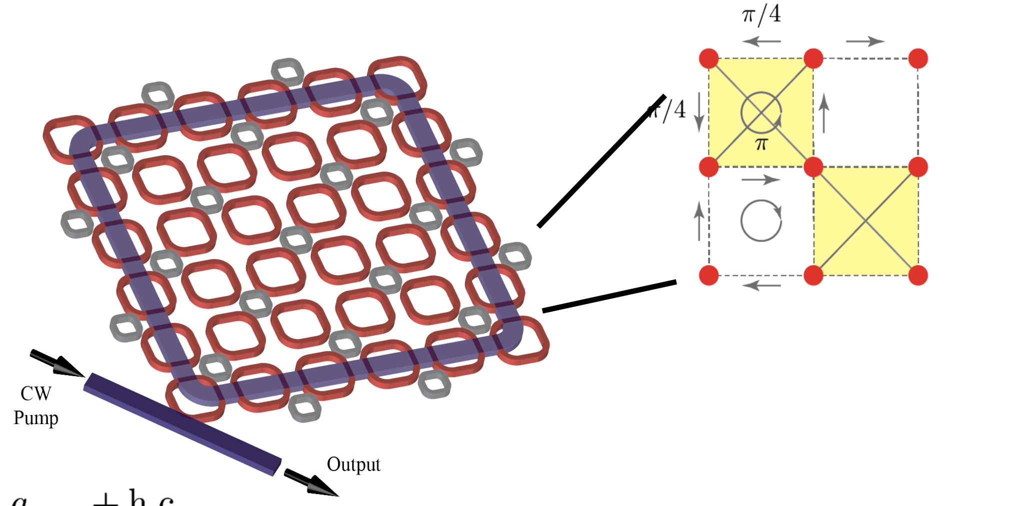
Nested phase matching
Science 2025

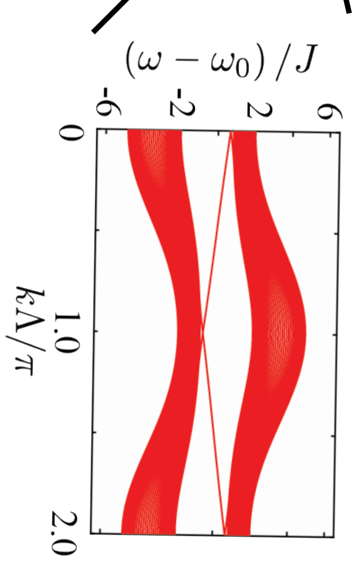
Quantum Metamorphosis and infinitely nested soliton
Under review in Nature

Ultrabroadband artificial gauge fields
In preparation

➢ Topological comb model



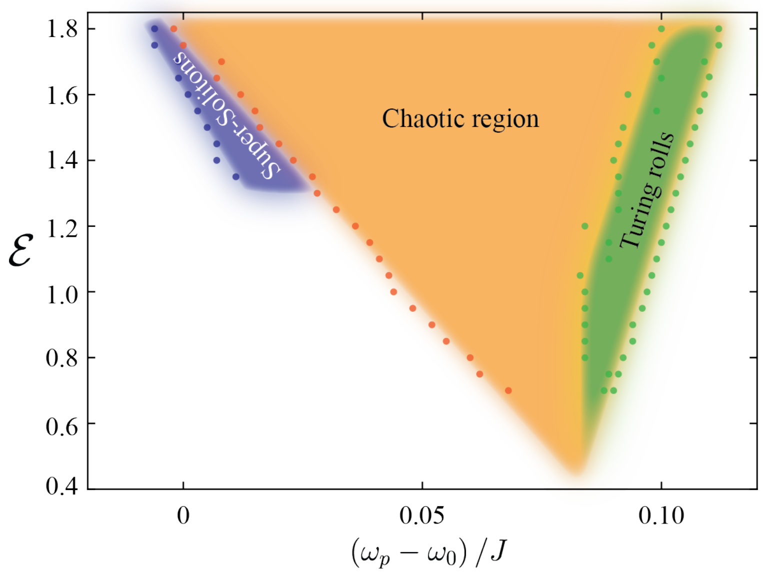


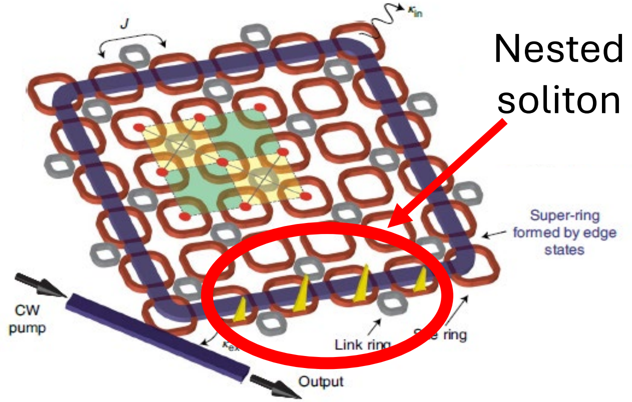


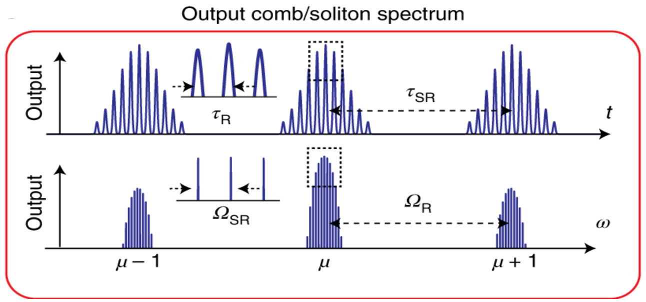
$$H_L = \sum_{m,\mu} \omega_{0,\mu}\, a^\dagger_{m,\mu} a_{m,\mu} - J \sum_{\langle m,n \rangle} a^\dagger_{m,\mu}\, a_{n,\mu} e^{-i\phi_{m,n}} - J \sum_{\langle\langle m,n \rangle\rangle} a^\dagger_{m,\mu}\, a_{m,\mu} + \text{h.c.}$$

$$-\beta \sum_{m,\mu} \hat{a}^\dagger_{m,\mu_1} \hat{a}^\dagger_{m,\mu_2} \hat{a}_{m,\mu 3} \hat{a}_{m,\mu 4} \delta_{\mu_1+\mu_2,\mu_3+\mu_4}$$

Solve driven-dissipative mean field dynamics





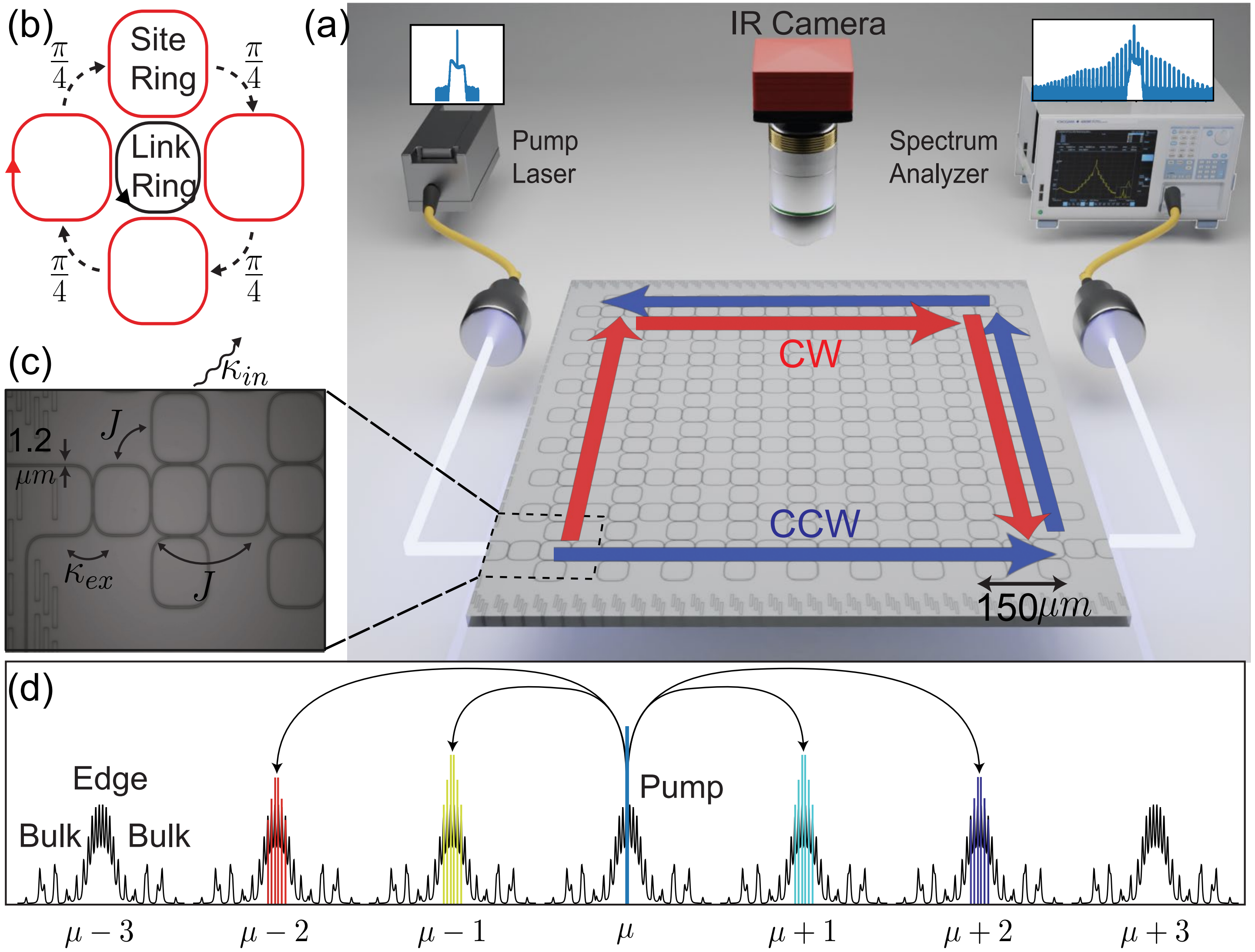


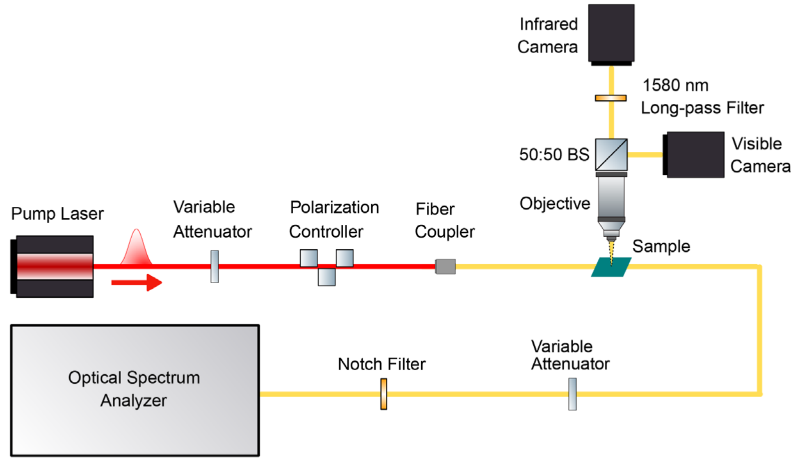
Fast time L/c
Slow time N/J

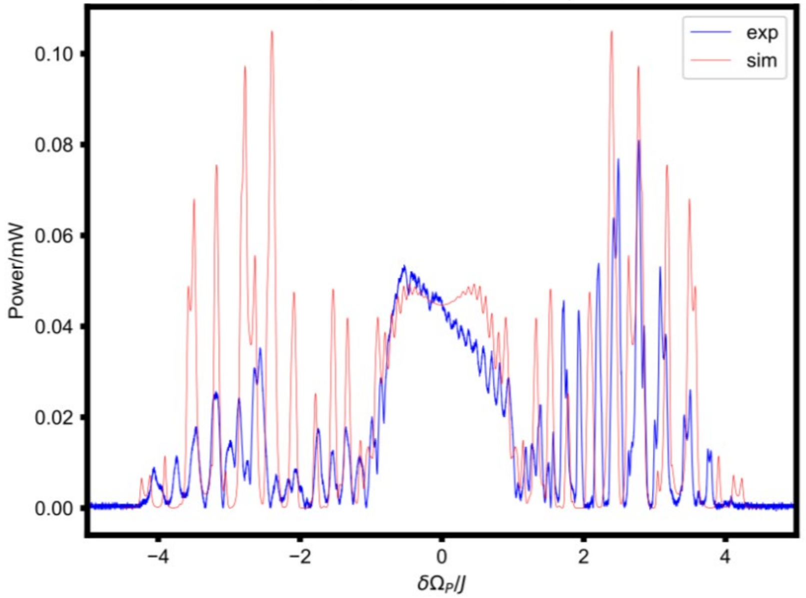
S. Mittal et al., Nature Physics (2021)

Mittal et al., PRL, 123, 043201 (2019)
Leykam et al., PRL, 121, 023901 (2018)

➢ Experimental realization of topological frequency combs



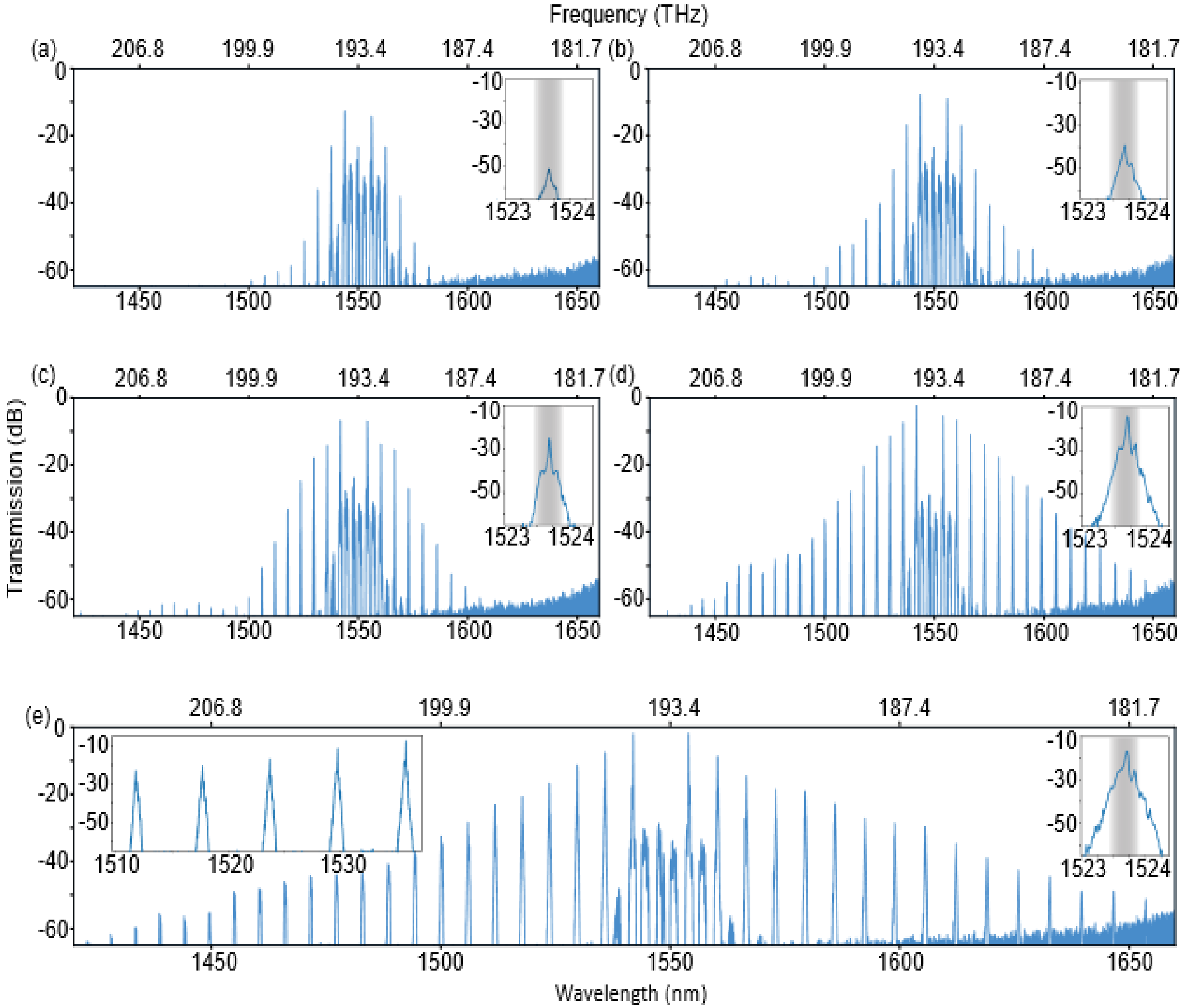




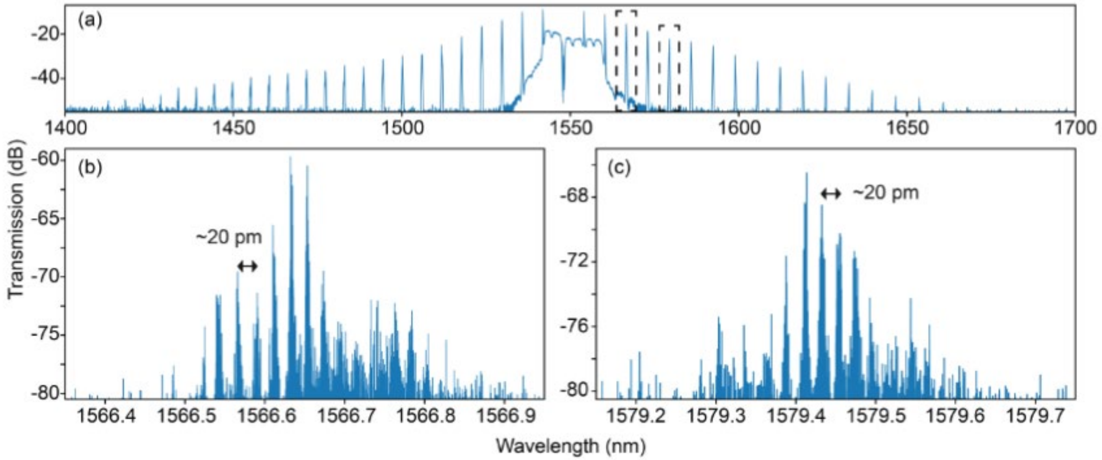
Linear drop spectrum

C. Flower, M. J. Mehrabad, L Xu et al. Science (2024)

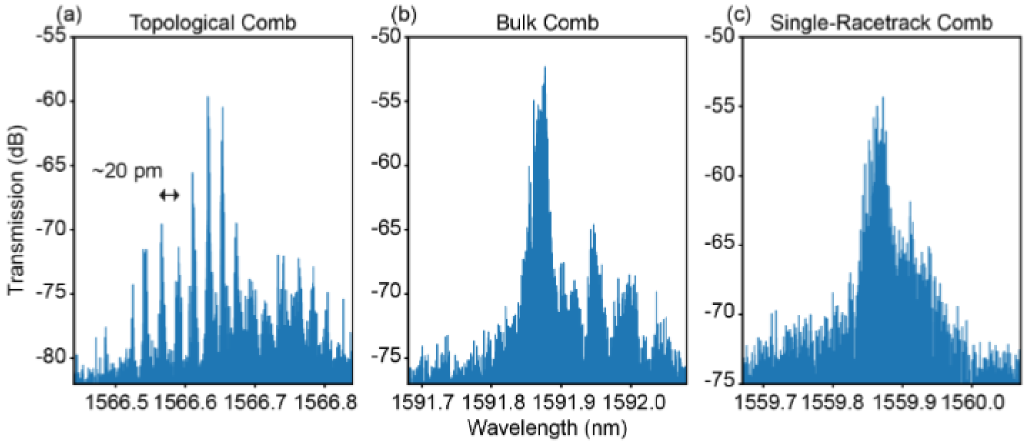
40pm grating based Optical Spectrum Analyzer (OSA): bandwidth ~1000nm. Can resolve single ring FSR of 6 nm but not super ring FSR of 20 pm.



0.04pm heterodyne based OSA: yay!

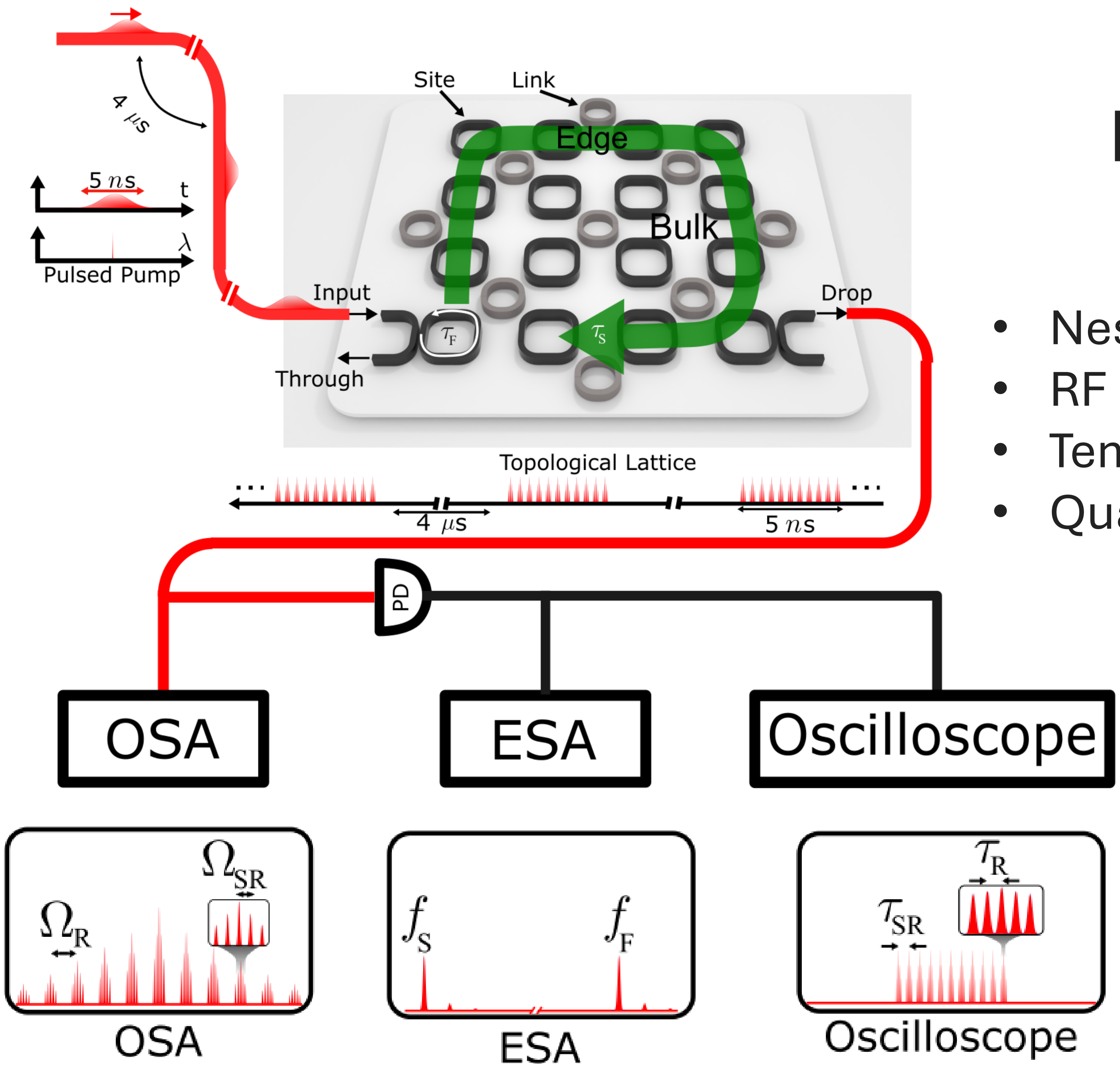


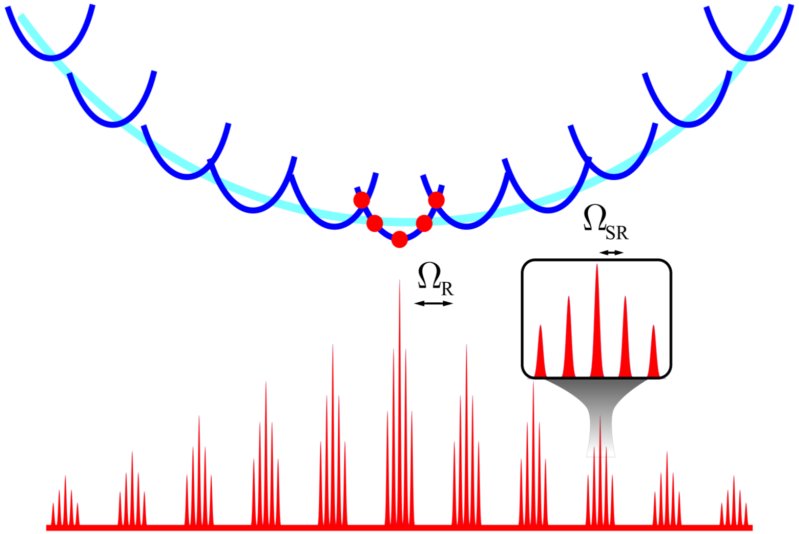
Pump the edge modes vs. pump the bulk modes vs. pump a single ring



Probing the photonic synchronization

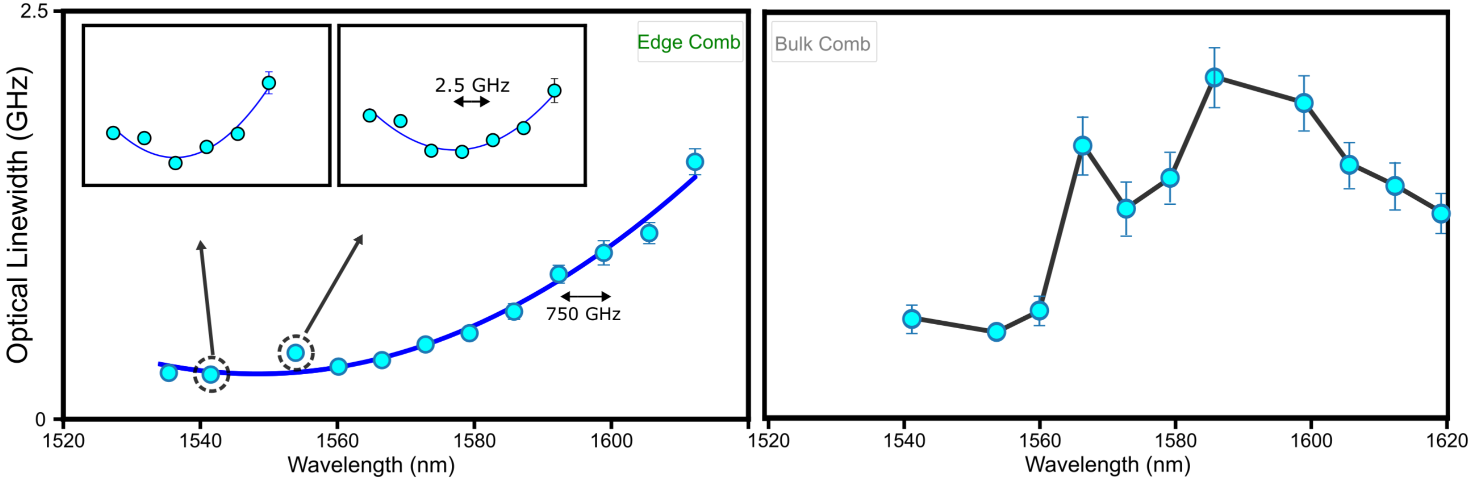
- Nested comb spectrum
- RF beat note
- Temporal pulses
- Quadratic linewidth distribution





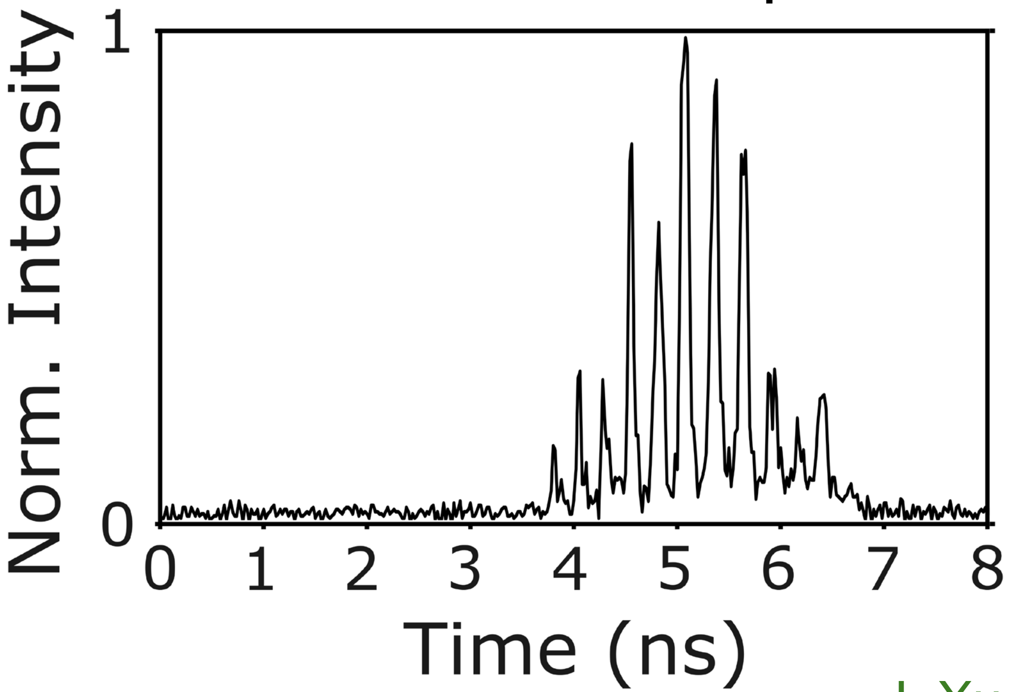
L Xu , M.J. Mehrabad, C.J. Flower et al. Science Advances (2025)

➢ **Quadratic linewidth distribution for both time scales**

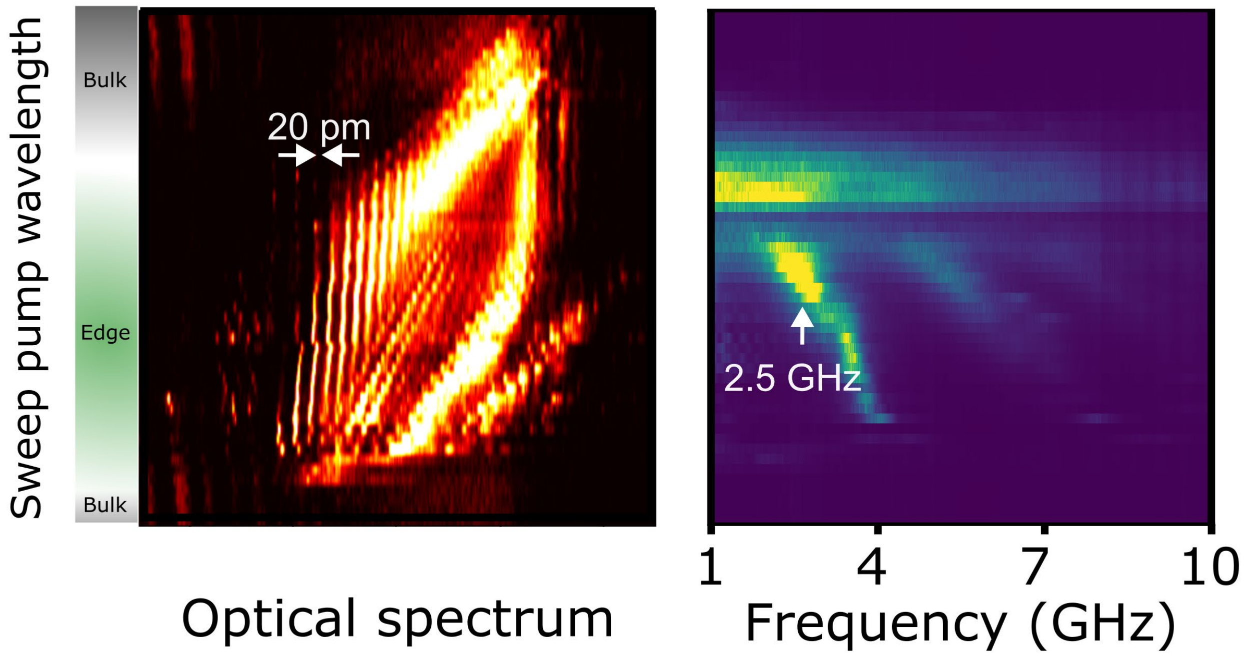


All measurements highly suggests mode locking for two timescales on topological edge combs

➢ **20 GHz Oscilloscope**



➢ **Nesting on OSA and RF beat note**



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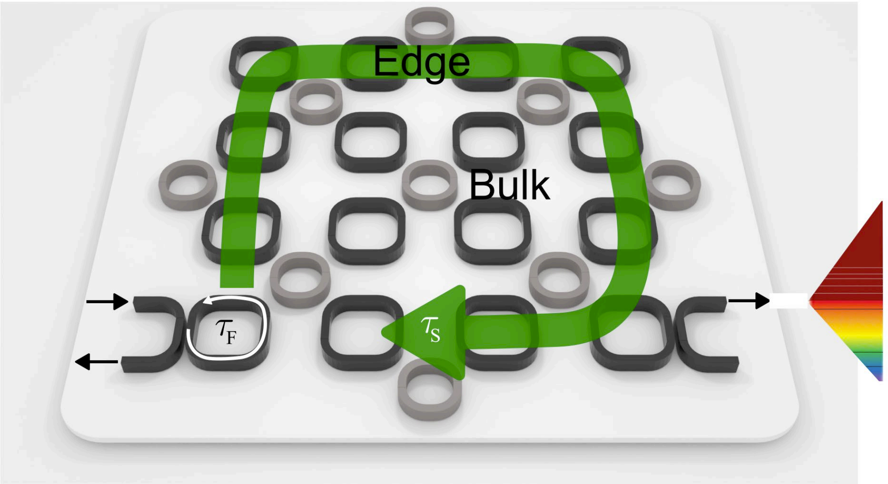
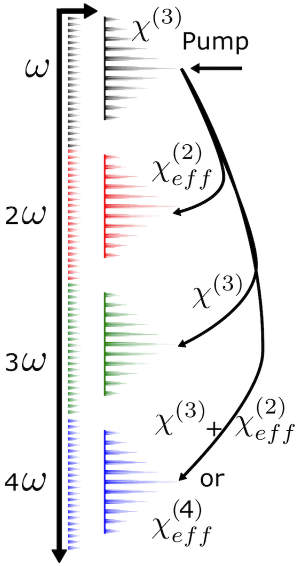
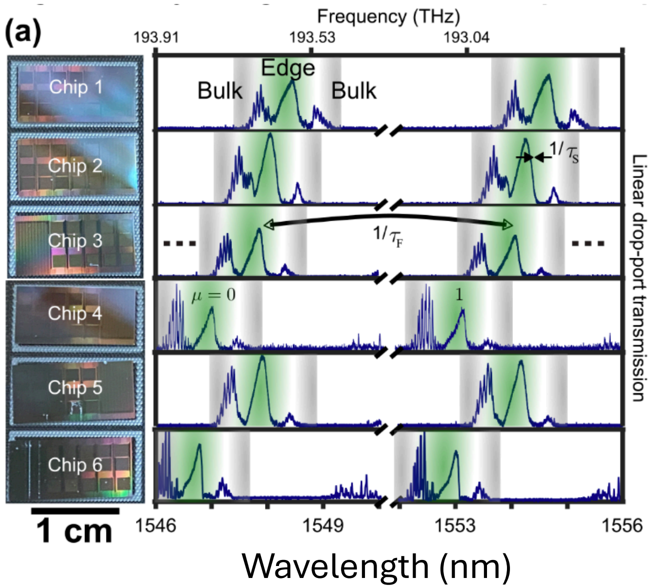
Surprising generation of harmonics



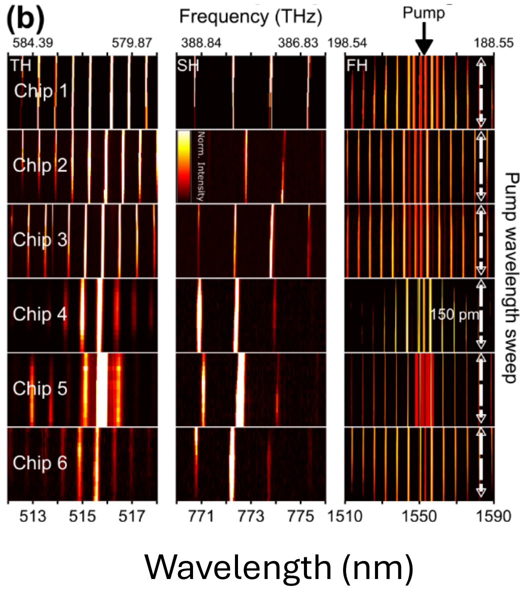


Same design, 6 copies



Optical Spectrum

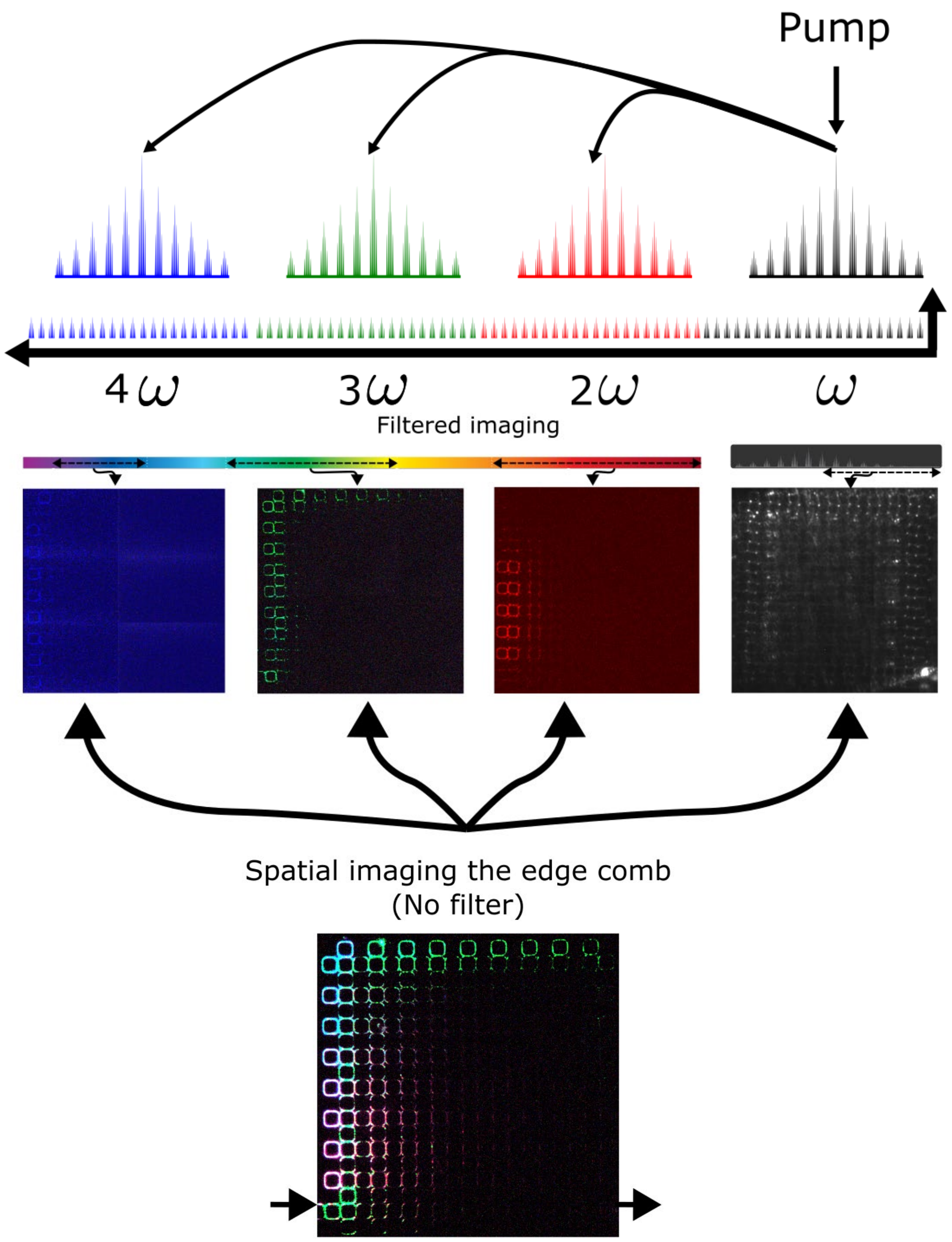
3ω 2ω ω



- First-ever observation of fourth harmonic in SiN
- Results with no optimized design or post-fabrication tuning
- 100% wafer-scale device yield

M.J. Mehrabad, L Xu et al. Science (2025)

Imaging the harmonics



All generated harmonics are confined to the edge of the system

Pump: 1550 nm

Red → SHG: 775 nm

Green → THG: 517 nm

Blue → THG: 388 nm

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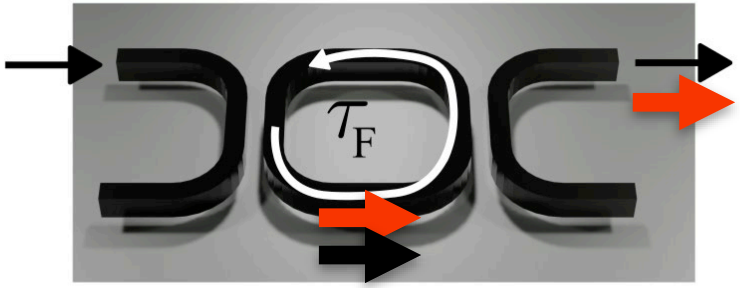
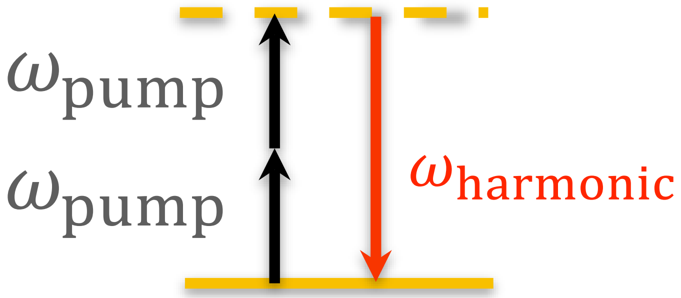
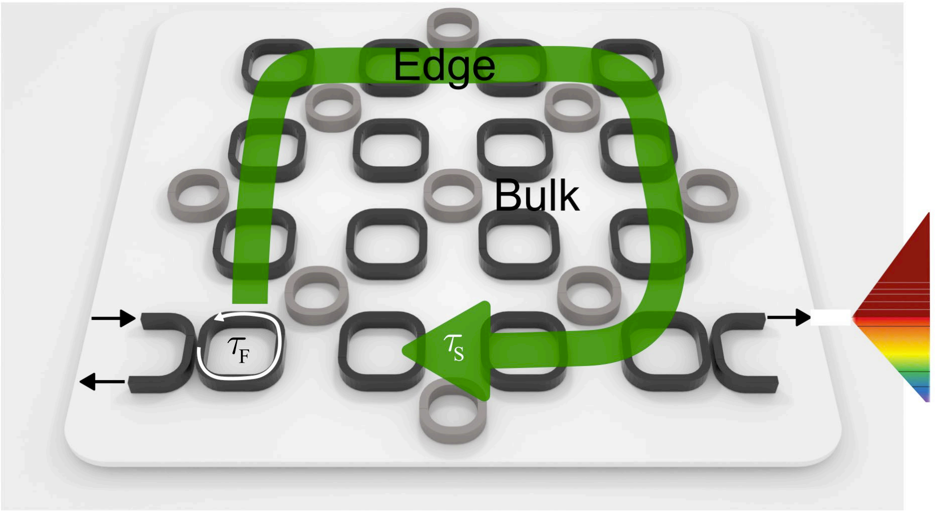
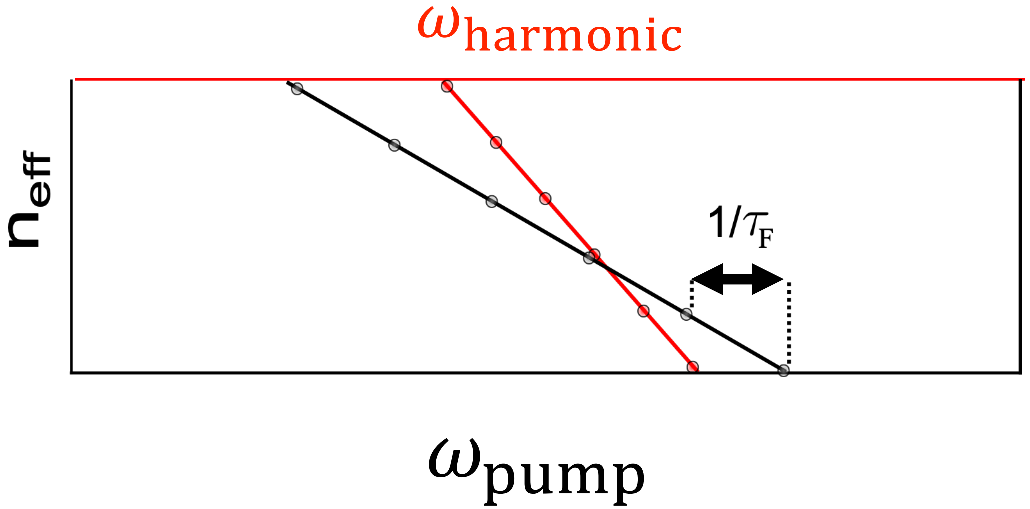
Is this a new phase matching mechanism?

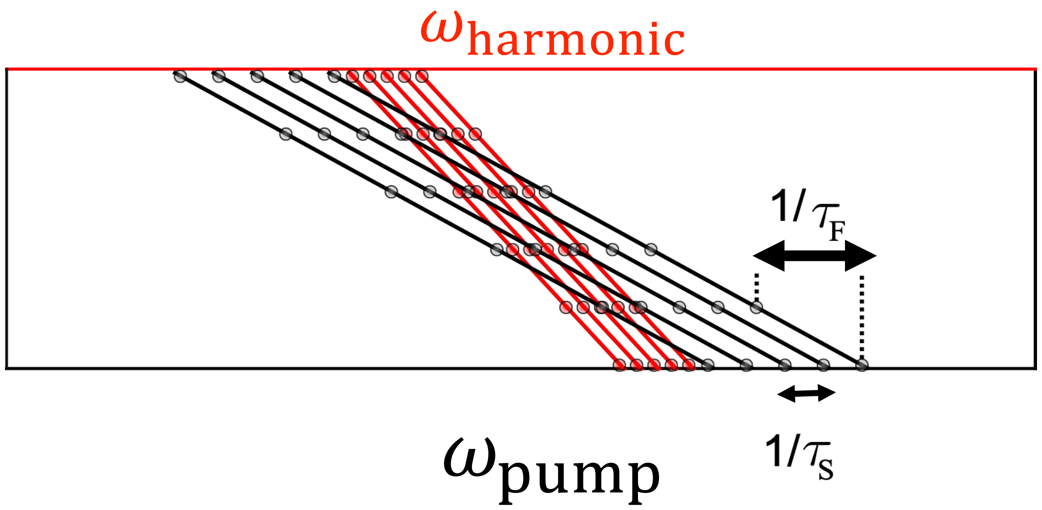




$$2k_{pump} = k_{harmonic}$$



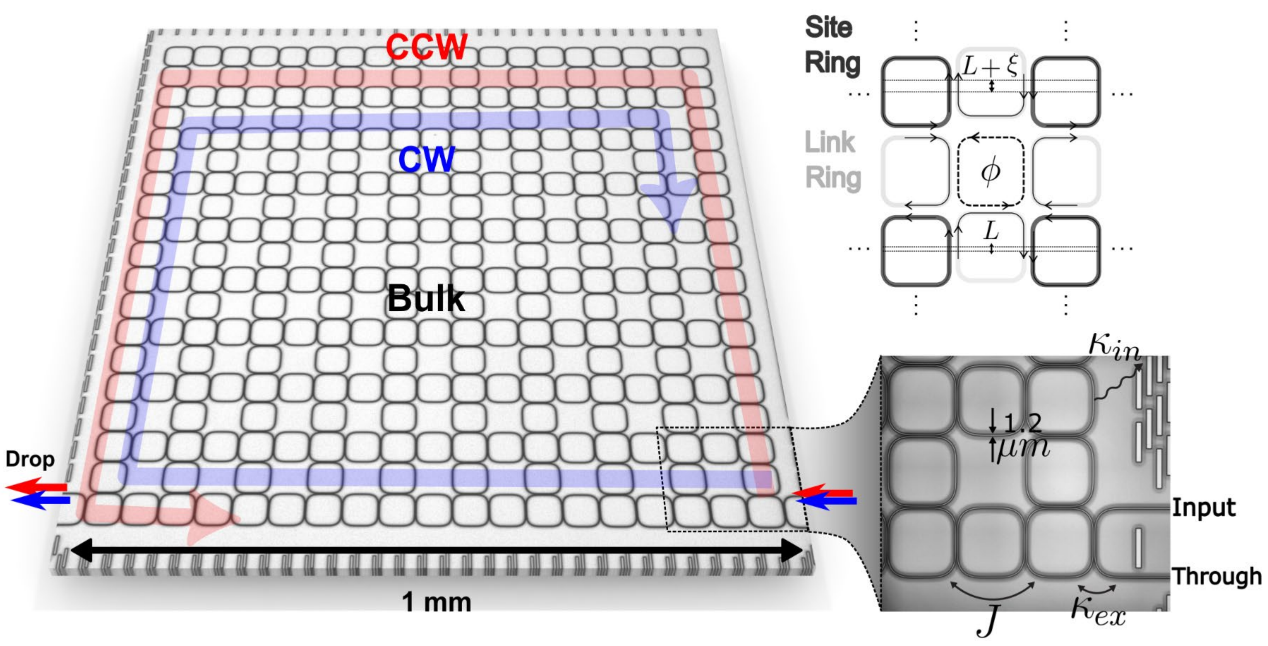


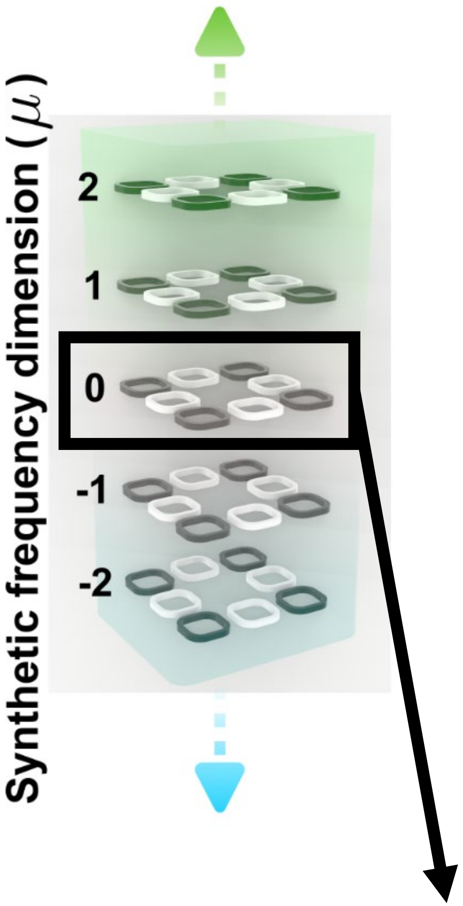


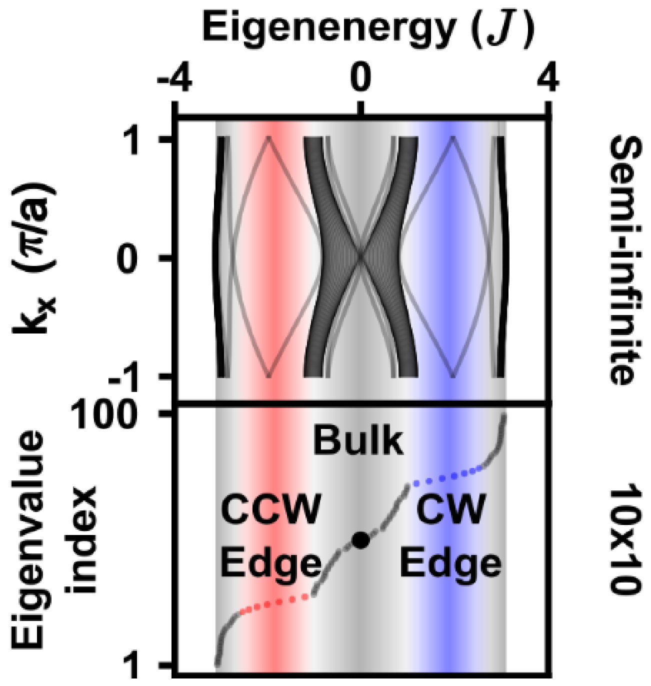
M.J. Mehrabad, L Xu et al. Science (2025)

New theoretical framework for nonlinear topological photonics

➢ The integer quantum Hall lattice







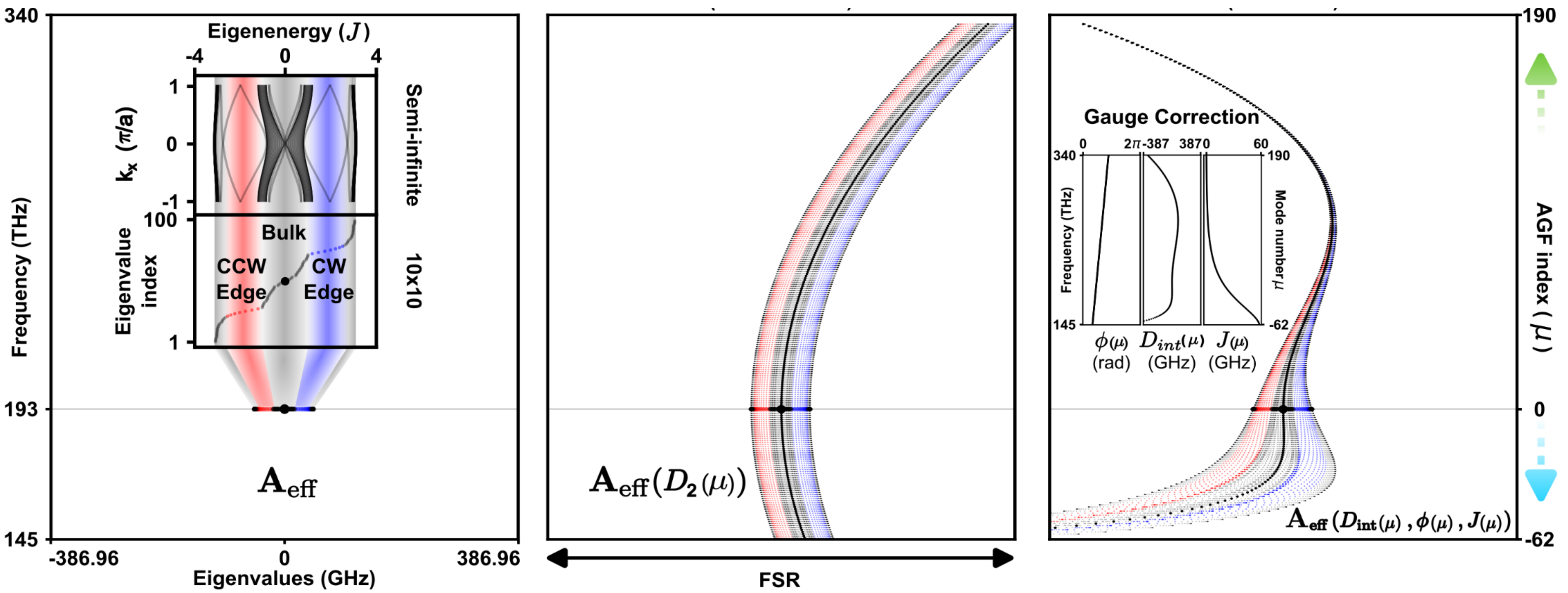
Narrow-band

Three parameters:
Onsite potential ω_0
Hopping strength J
Gauge field $\mathbf{A} = (y\phi, 0, 0)$

$$H_{\mathrm{IQHE}} = \sum_{x,y} \omega_0 \hat{a}^\dagger_{x,y}\,\hat{a}_{x,y} - J\sum_{x,y}\left(\hat{a}^\dagger_{x+1,y}\,\hat{a}_{x,y}\,e^{-i\,y\,\phi} + \hat{a}^\dagger_{x,y+1}\,\hat{a}_{x,y} + \mathrm{h.c.}\right).$$

L Xu, A Padhye et al. In prep

Broadband multimodal tight-binding approximation?



Previous narrow-band topological photonics works

Previous broad-band multimodal works

This work: new tight-binding framework

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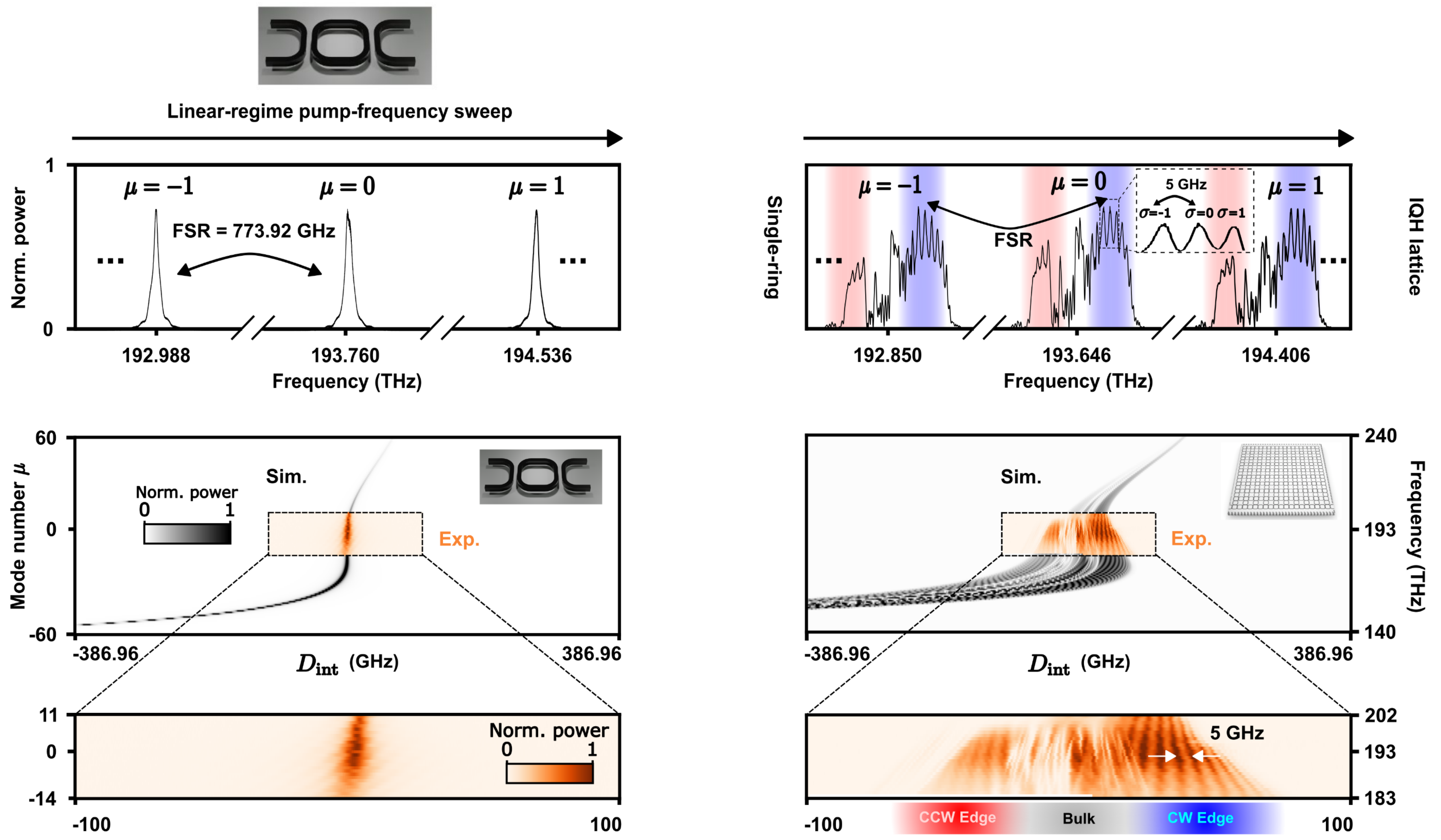
Multi-shot probing of the dispersive tight-binding Hamiltonian



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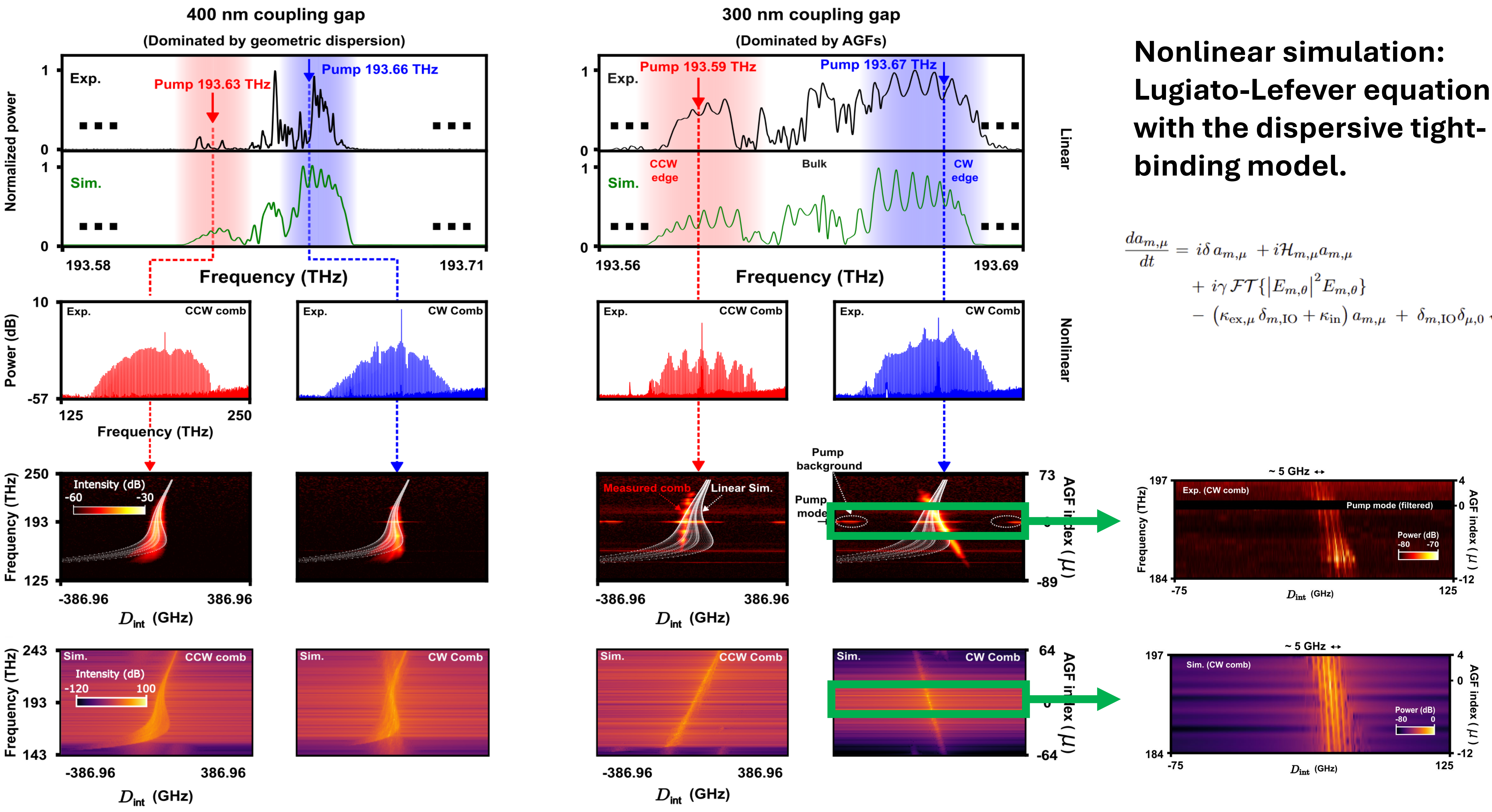
Nonlinearity to single-shot probing the dispersive tight-binding model



Nonlinear simulation: Lugiato-Lefever equations with the dispersive tight-binding model.

$$\frac{da_{m,\mu}}{dt} = i\delta\, a_{m,\mu} + i\mathcal{H}_{m,\mu}a_{m,\mu} + i\gamma\,\mathcal{FT}\{|E_{m,\theta}|^2 E_{m,\theta}\} - (\kappa_{\mathrm{ex},\mu}\,\delta_{m,\mathrm{IO}} + \kappa_{\mathrm{in}})\,a_{m,\mu} + \delta_{m,\mathrm{IO}}\delta_{\mu,0}\sqrt{2\kappa_{\mathrm{ex},\mu}}\mathcal{E}$$

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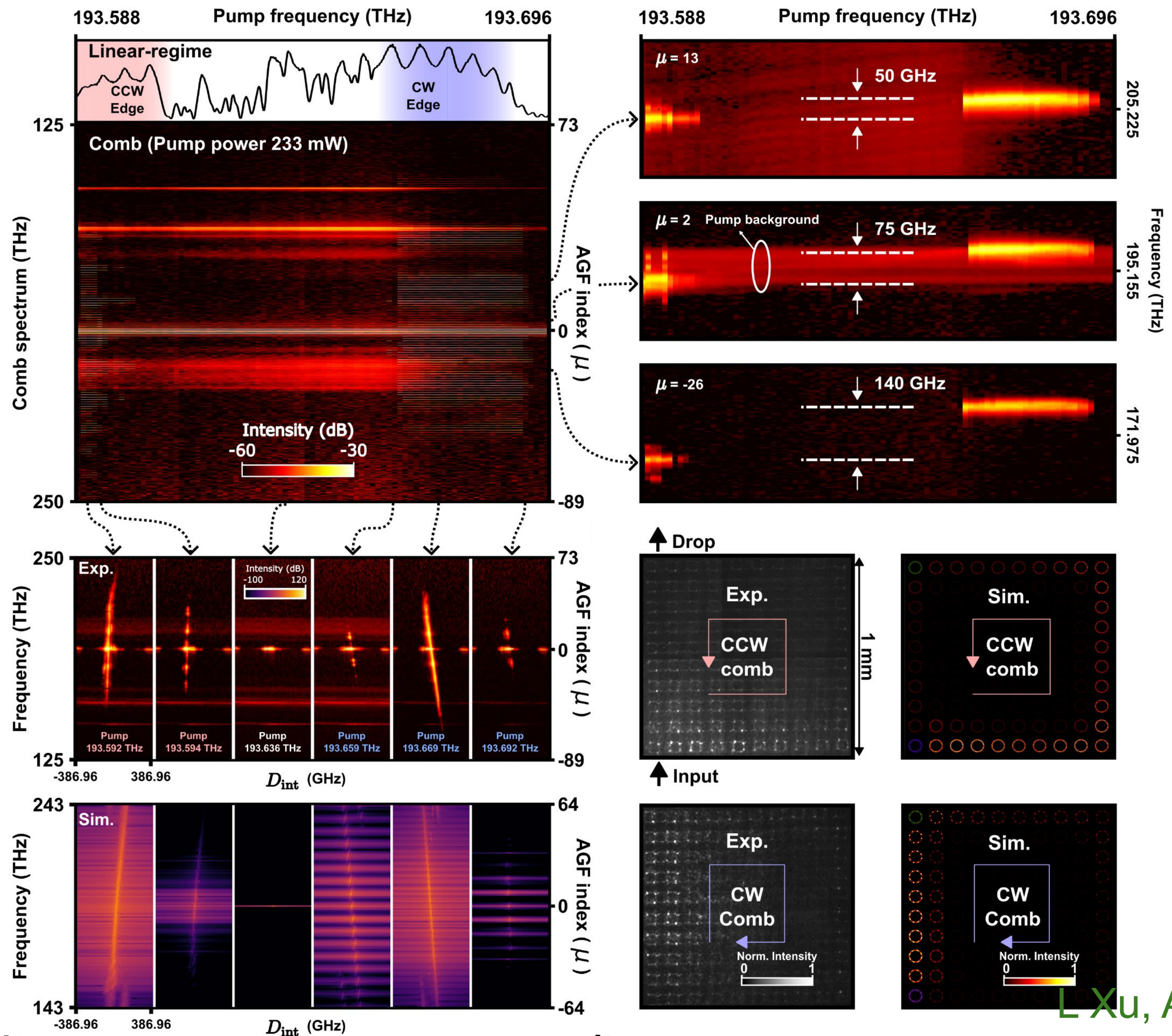
➢ Topological protection survives with strong nonlinearity!



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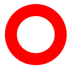
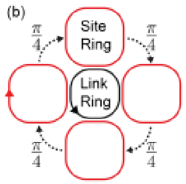
➢ Infinite layers of quantum hall topologies?

Single AQH Plaquette



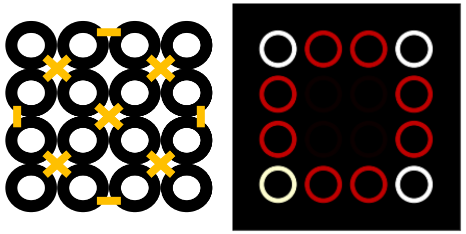


More Is Different

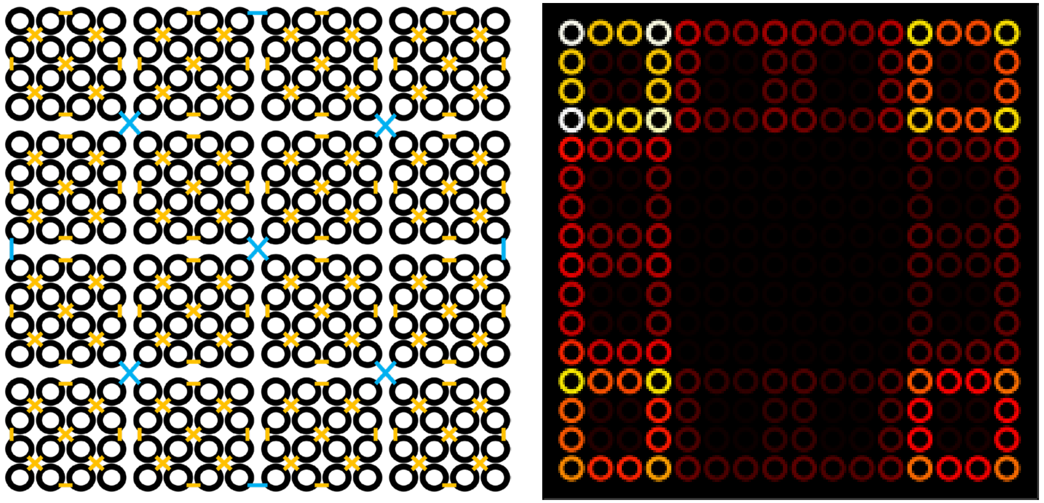
Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

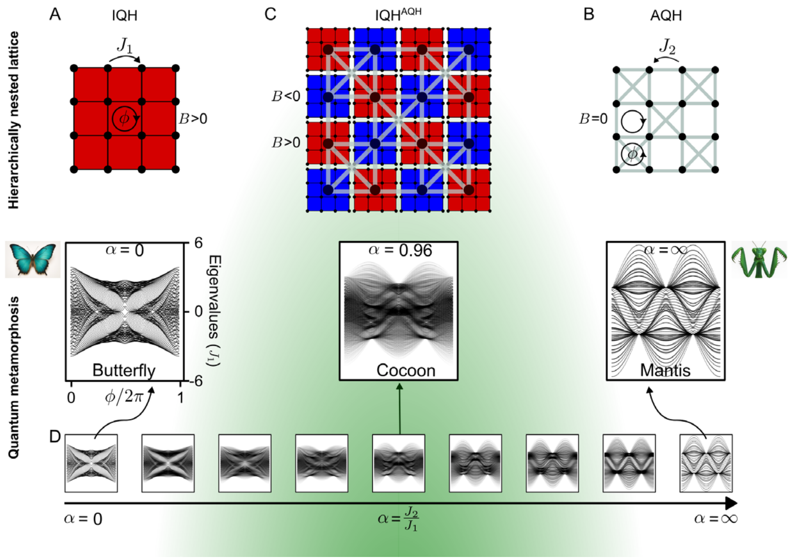
AQH Lattice

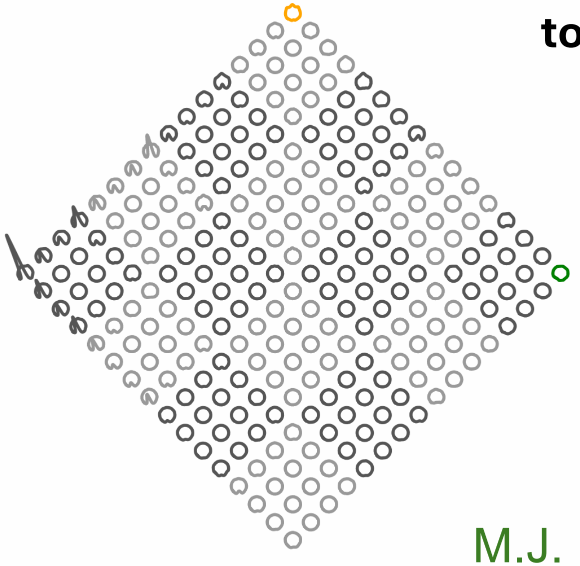


2nd Order Lattice

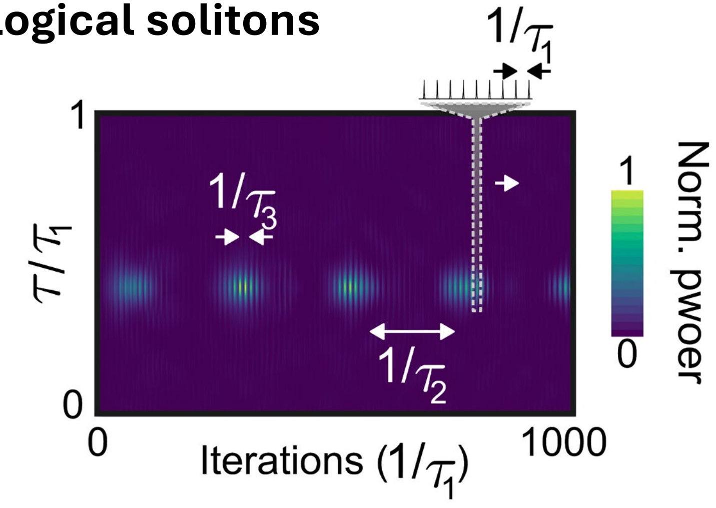


More than simple addition! New topological bands: isolated flat bands, magic flat edge bands etc…

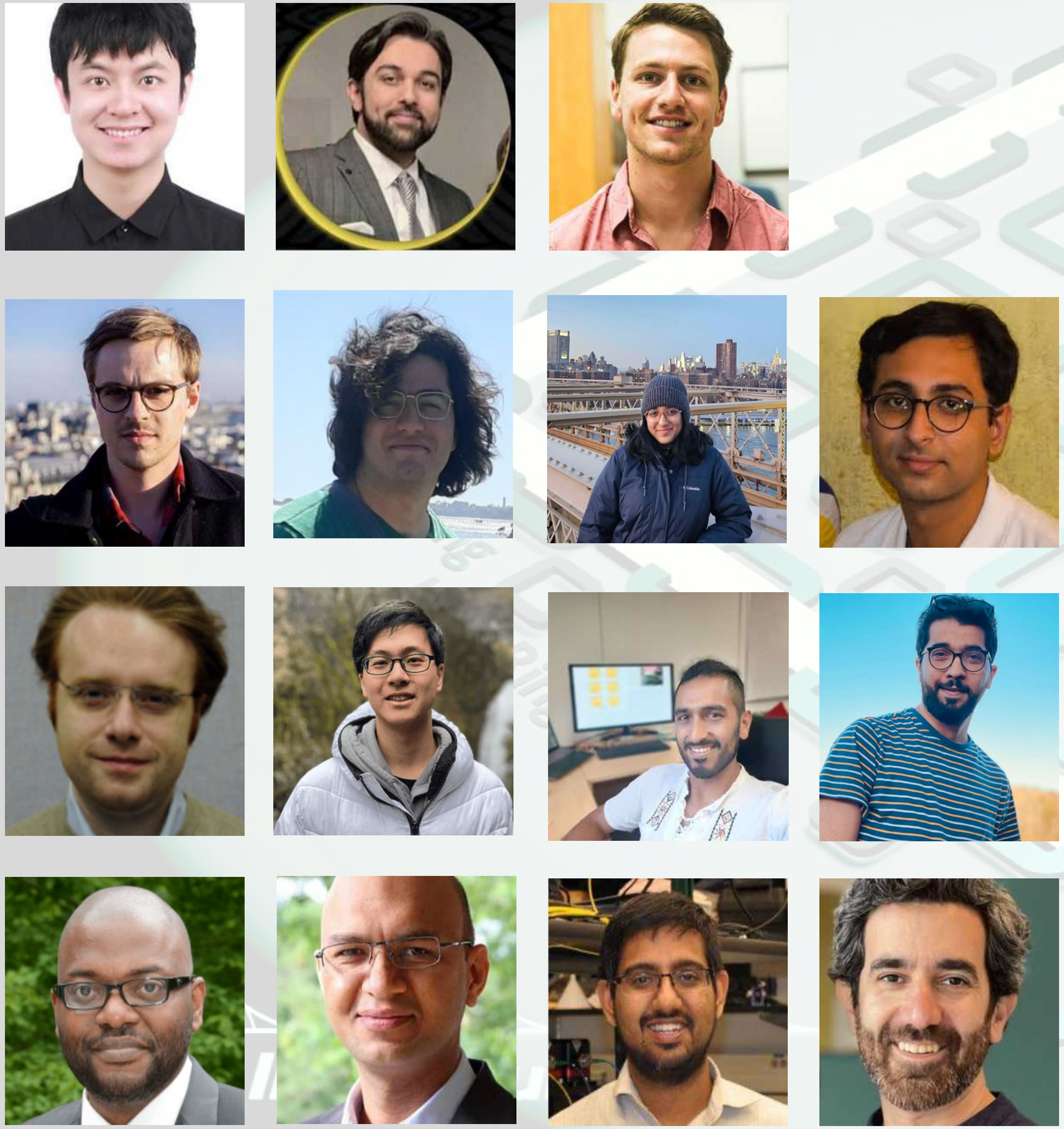




Exotic higher-order topological solitons



M.J. Mehrabad, A. Parhizkar, L Xu et al. Submitted



Thank you for your time!

Contact: lidaxu66@umd.edu

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DEPARTMENT OF PHYSICS

NIST
NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
U.S. DEPARTMENT OF COMMERCE

Northeastern University
College of Engineering

The elastic tape model

- The frequency of a soliton comb line is:

$$\omega_\mu(t) = \omega_0(t) + \mu D_1(t), \qquad \mu \in \mathbb{Z},$$

- Therefore, a fluctuation of each teeth is:

$$\delta\omega_\mu(t) = \delta\omega_0(t) + \mu\,\delta D_1(t).$$

- The power spectral density of any comb teeth is:

$$\boxed{S_{\delta\omega_\mu}(f) = S_{\delta\omega_0}(f) + \mu^2\, S_{\delta D_1}(f) + 2\mu\,\Re\{S_\times(f)\}\,.}$$

- Linewidth becomes quadratic as a function of \mu.

$$\Gamma_\mu = \pi\, S_{\delta\omega_\mu}^{(\mathrm{white})} \;=\; \Gamma_0 + 2\mu\,\Gamma_\times + \mu^2\,\Gamma_{\mathrm{rep}},$$

- If not soliton comb:

$$\omega_\mu = \omega_0 + \mu D_1 + \tfrac{1}{2}\mu^2 D_2 + \tfrac{1}{6}\mu^3 D_3 + \cdots\,.$$

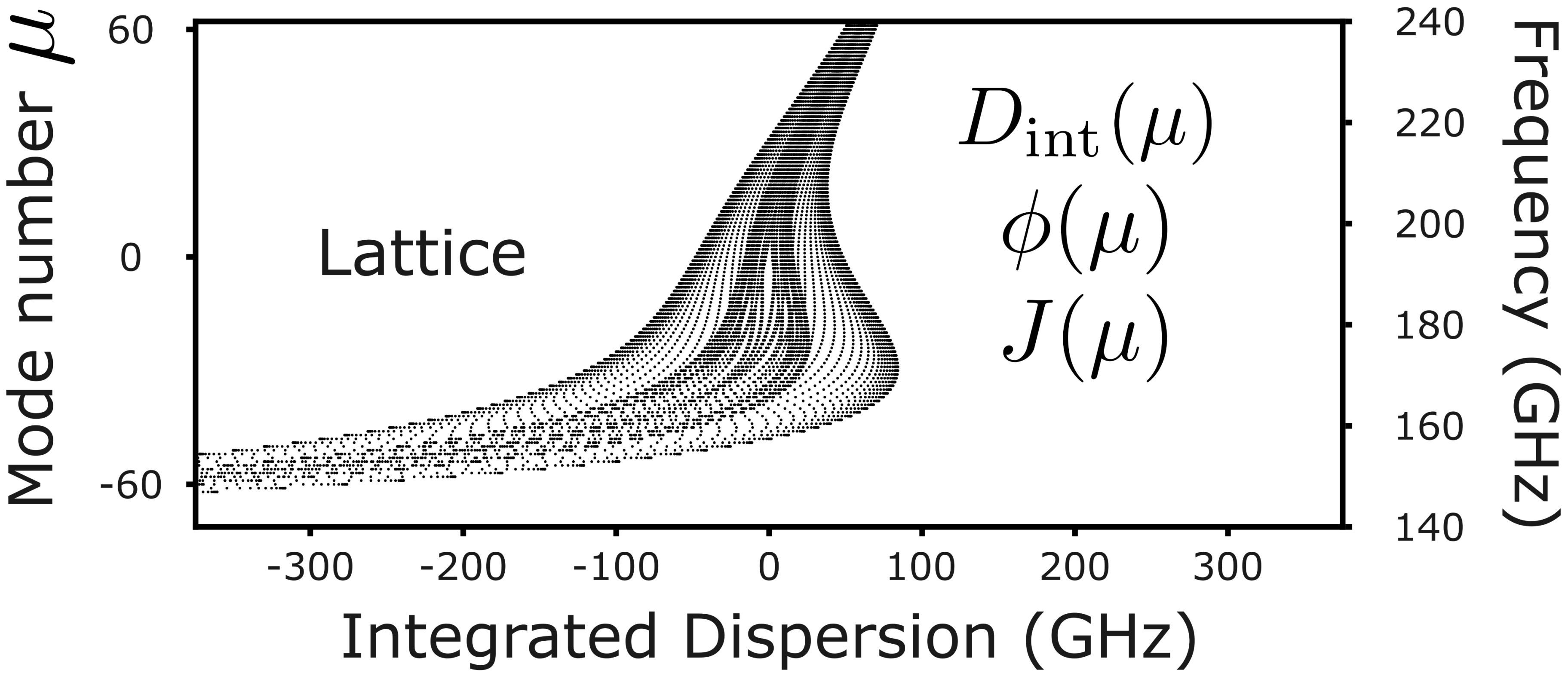
- Then:

$$S_{\delta\omega_\mu}(f) = S_{00} + \mu^2 S_{11} + \tfrac{1}{4}\mu^4 S_{22} + \tfrac{1}{36}\mu^6 S_{33}$$
$$+\,2\mu\,\Re S_{01} + \mu^2\,\Re S_{02} + \mu^3\,\Re S_{03} + \mu^3\,\Re S_{12} + \cdots\,.$$

Victor Torres-Company, Nature Communications (2022)

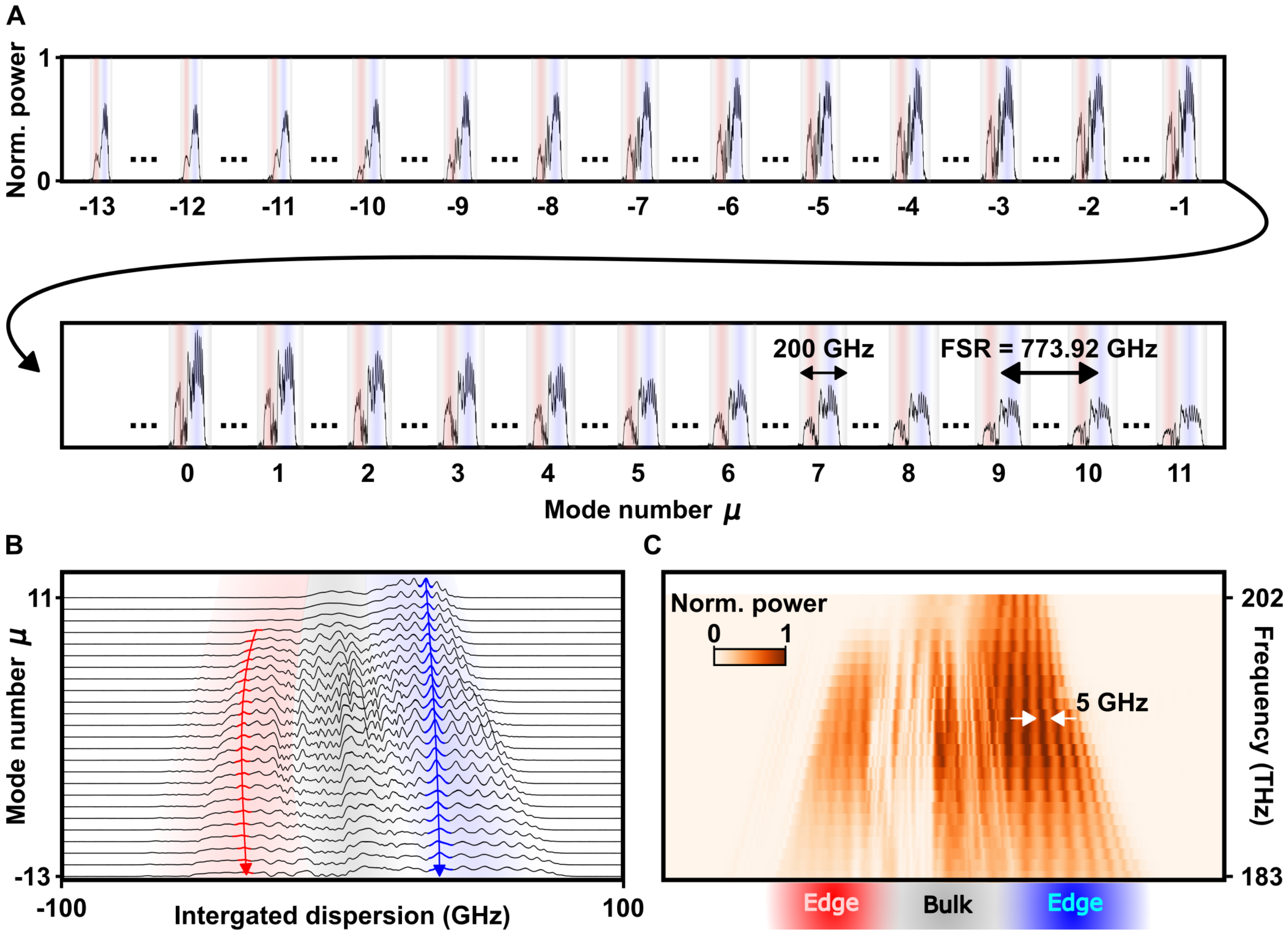
The dispersive tight-binding Hamiltonian

$$H_{\mathrm{IQHE}}(\mu) = \sum_{x,y} \{\omega_\mu - J(\mu)\cot[\zeta\phi(\mu)]\}\, \hat{a}^\dagger_{x,y}\,\hat{a}_{x,y} - J(\mu)/\sin[\zeta\phi(\mu)] \sum_{x,y}\left(\hat{a}^\dagger_{x+1,y}\,\hat{a}_{x,y}\,e^{-i\,y\,\phi(\mu)} + \hat{a}^\dagger_{x,y+1}\,\hat{a}_{x,y} + \text{h.c.}\right)$$

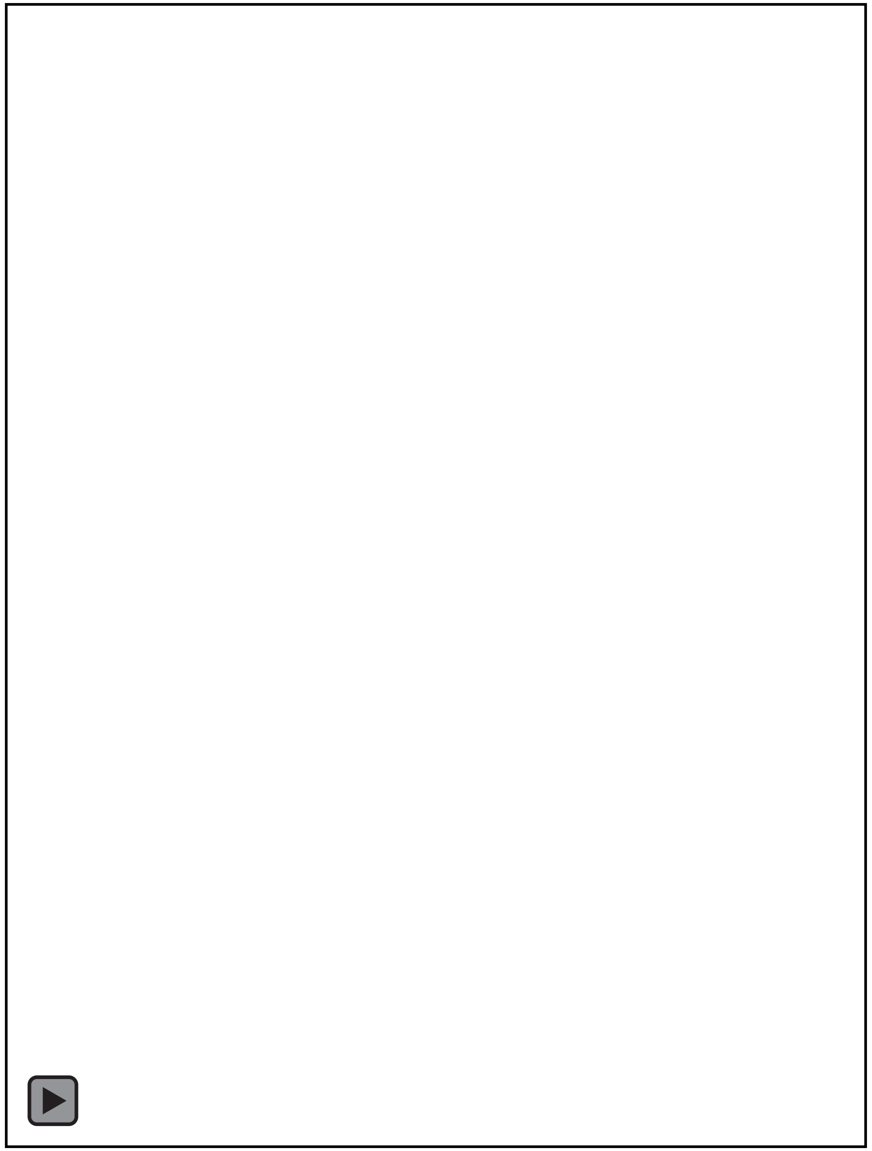


L Xu, A Padhye et al. In prep

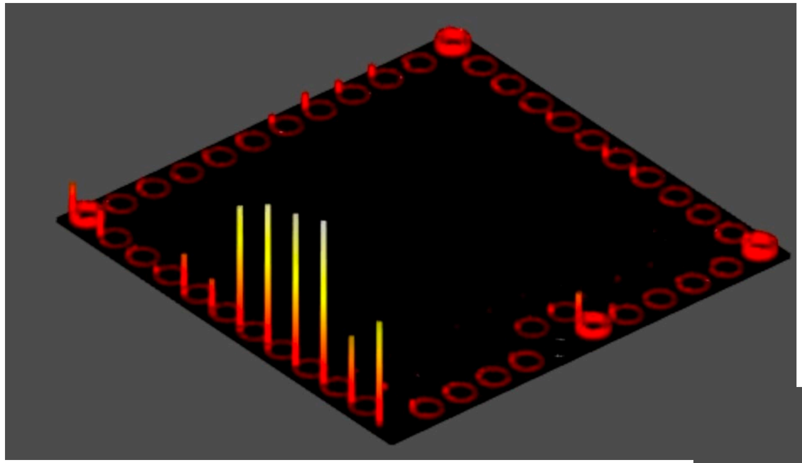
Fig SI: Constructing the integrated dispersion from linear-sweep measurement

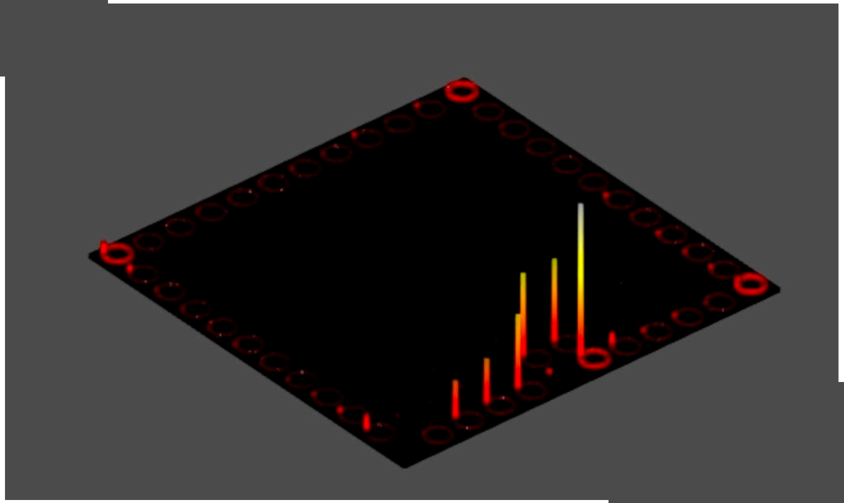


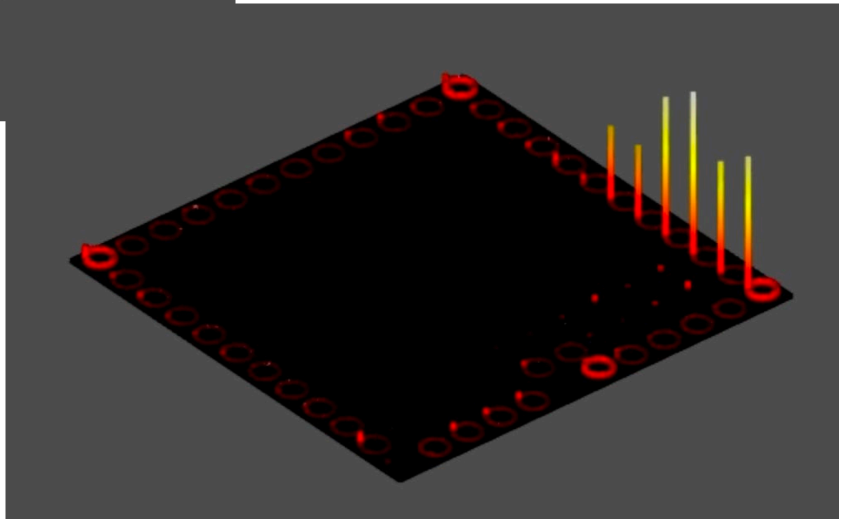
- **Nested solitons**

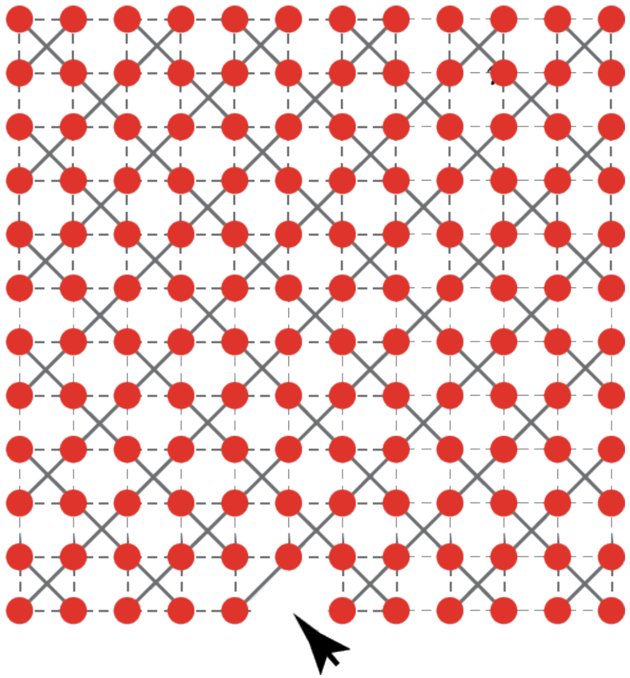


➢ Robustness against defects



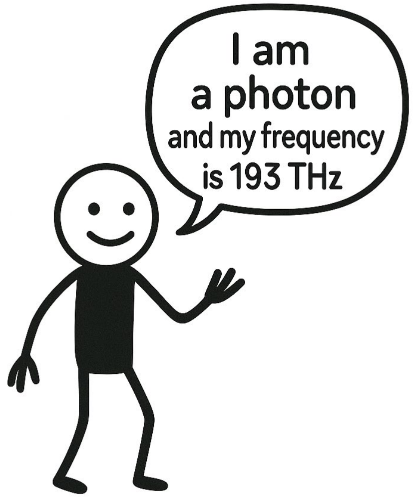






1

➢ Topological mode-locking: a fairy tale



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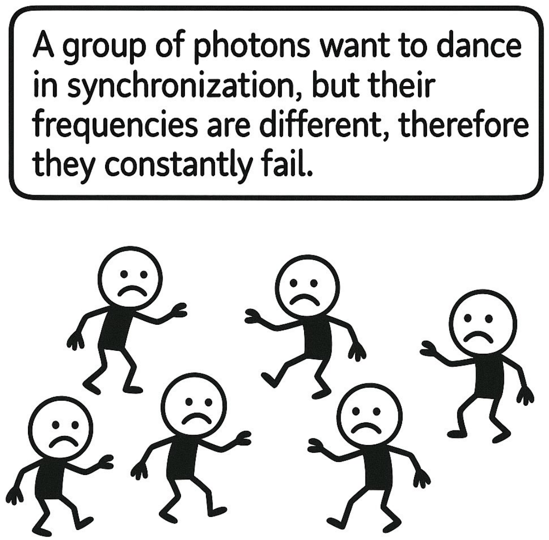

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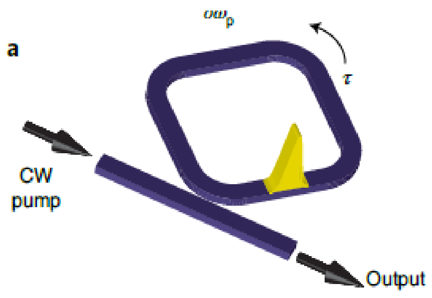

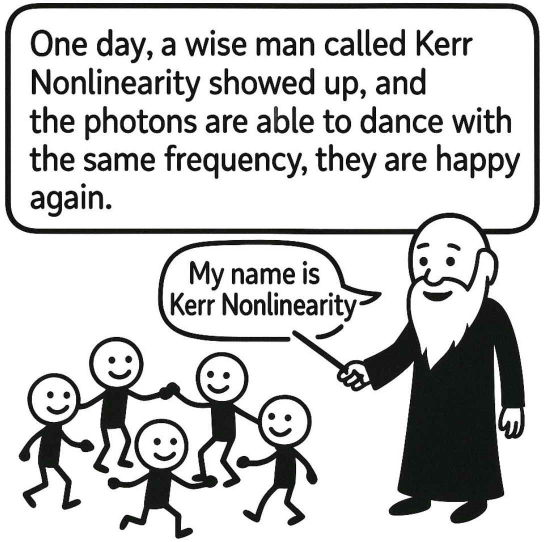

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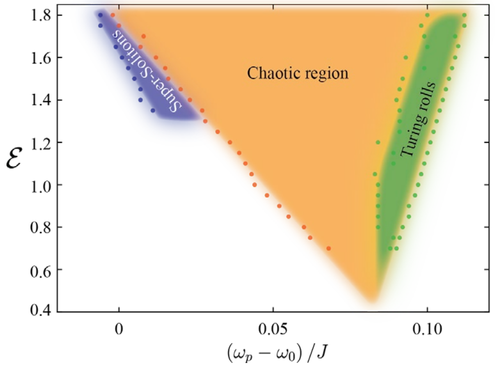
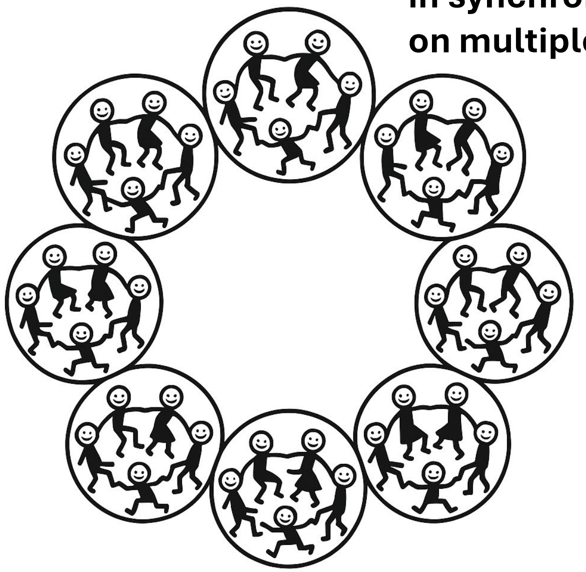
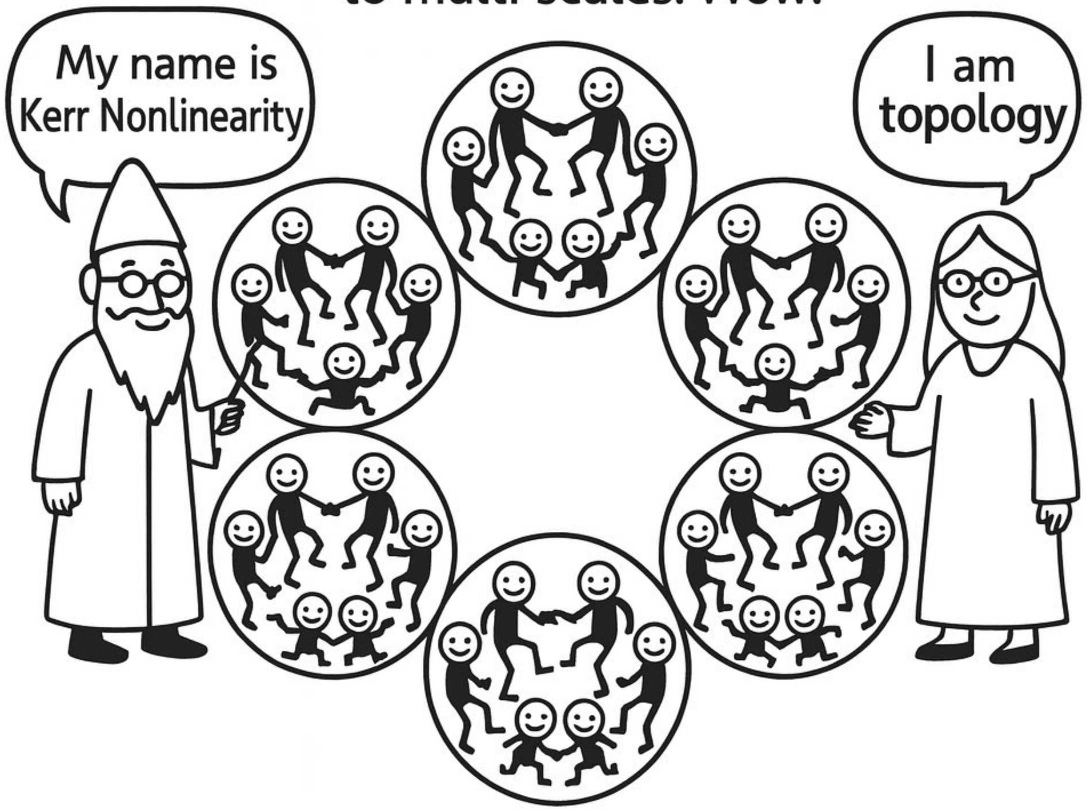


Can photons dance in synchronization on multiple scales?



Later, the photons get to know topology, who helped them improve their dance to multi scales. Wow!



The photons dance happily ever after.